Hermeneutical Reflections on Mathematical Significance

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Abstract: This paper aims to investigate the meaning of mathematics in context of its applicability. Realism and Nominalism as dominant traditional ways of philosophizing are critically analyzed with the end of working out the problems that would lead to the priority of the question of applicability over abstract theorizing about metaphysical existence of mathematical entities. An answer to the question of applicability will be examined to the extent which would lead to an analogy with historical practices of classical philology. Interpretive historical inquiry will be made to further extend analogy between the role of mathematics in context of its descriptive applicability in natural sciences, and the role of Dilthey’s hermeneutics as universal methodology for human sciences. The purpose of this investigation is to enrich conventional conceptions of mathematics within the sphere of mathematics education to incorporate a possibility of a hermeneutic approach to mathematical pedagogy.

Keywords: Hermeneutics, Realism, Nominalism, Philology, Mathematics Education.

1. Introduction: Metaphysical Problematics of Realism vs. Nominalism

Any investigation about the reference or meaning of mathematical entities and their systematic relationship traditionally involves taking at least either a realist or a nominalist stance. As model example, one can take Platonism as realism which amounts to a doctrine that admits the independent existence of mathematical objects and their systematic relation (Dummet, 1991, p. 301). Then nominalism can simply be its opposite that denies such an independent existence. However, there are exceptions or significant deviations from these two. For instance, Brouwer’s intuitionism shouldn’t be classified as being either a realist or nominalist. On the other hand Hilbert’s formalistic approach can be given a nominalistic twist by showing how it can help in nominalizing physical theories (Fields, 2016, p. 42ff)

Most of these traditional philosophical debates revolve around the nature of existence of mathematical objects and the possibility of working out the minimal conditions that can render statements about them to be true. Thus, most of these philosophical considerations either ignore or take the problem of applicability of mathematics only derivatively. On the other hand, as far as phenomenological aspect of the use of mathematics in theoretical sciences (especially theoretical physics) is concerned, as Mark Steiner rightly identifies with reference to some of the most respectable physicists of our times, the situation is quite the opposite (Steiner, 1998, p. 13ff). It was in this context that a debate started with Quine and Putnam (both as Platonists) about the essential indispensability of mathematics for science reached its pivot when Hartry Field provocatively denied this indispensability with his nominalistic account, which can be dubbed representational fictionalism, of how mathematics is applicable. According to this, mathematical theories are very much like fictions which help represent the world and thus constitute one among many options for describing it (Field, 1989).

However, neither any version of Platonistic realism satisfactorily answers the problem of applicability nor any nominalistic account sufficiently explains how mathematics helps render naturalistic theoretic explanations possible. Their mutual critiques as outlined by Mark Balaguer reaches a dilemma: whether abstract entities exist or they don’t exist, which requires a hermeneutic twist by showing how it can help in nominalizing physical theories (Field, 2016, p. 42ff).

2. On Descriptive Applicability of Mathematics

When the debate between nominalism and realism is pushed to its limits, I propose that the most fruitful point of contention is not the nature of existence of abstract mathematical objects (e.g. points, sets, numbers, ...etc.) but the question of indispensability of mathematics in context of its application. This shift of focus is concomitant with author’s hermeneutic and phenomenological convictions which prioritize interpretation and description over abstract theorizing, for it is in the use of mathematics (both inside and outside of mathematics) that one should start questioning the very sense of what mathematics is all about instead of first trying to seek the foundational insights through regional ontology and then help make sense of mathematical activity. Both Steiner and Fields realize the significance of this shift of focus. Both reach, however covertly, towards the hermeneutic turn that I intend to propose, but without actually making the turn. Fields attempts to see mathematical theories as fictions by nominalizing both physical and mathematical theories using conservative principle (which I find a modern avatar or interpretation of classic Okham’s razor) helps sus-
pend the judgment about an independent metaphysical existence of mathematical realm that has a deep craving in our natural attitude towards the meaning of mathematics. However, the suspension is not sufficient and requires a force to counter the possibility of its own nominalist residue.\(^4\) Steiner’s identification of seeing mathematical applications in theoretic physics with covert-overt anthropocentric values to the extent that mathematics becomes a particular anthropocentrism, i.e., a way of privileging human perspective, has interpretive (thus hermeneutic) underpinnings which can suffice for the above suspension of judgment that I find a phenomenological requirement to shift towards the proposed turn. According to him, what characterizes a mathematical activity is ultimately not some kind of Fregean semantics but beauty and convenience (Steiner, 1998, pp. 17, 15, 72ff, 115). On the other hand, his account of nth-order formalist and non-formalist Pythagorean mathematical analogy\(^5\) in context of most of the modern applications in theoretical physics, which he classified under the rubric of Pythagoreanism, seems to have close parallels with interpretive schemata as practiced in classical philology, though the latter had the aim to identify authentic from inauthentic texts in classical Greek literature, besides being an aid to reconcile conflicting interpretations (Dilthey, 1996b). In order to argue this parallelism,\(^6\) first consider Steiner’s account.\(^7\) It roughly amounts to a relationship R known to be existent between two mathematical objects (or structures), say X\(_1\) and X\(_2\). Suppose, there are two physical objects (or phenomena), say Y\(_1\) and Y\(_2\), already known to be physical interpretations of X\(_1\) and X\(_2\) respectively. Then even if there is no physical warrant of any physical relation between Y\(_1\) and Y\(_2\), a warrant would be imposed as a physical avatar of R. In fact, even if there is no known Y\(_2\) as a physical interpretation of X\(_2\), a physical interpretation can be conjectured as natural entity or phenomenon to exist that would hold a similar relation R between Y\(_1\) and Y\(_2\). Steiner’s historical account of Maxwell’s ‘displacement current’ facilitated such a warrant to explain and predict Maxwell’s electromagnetic radiation. Similarly, on philological side, in order to resolve the conflict of interpretation over authorial intention and the authenticity of authorship, both Zenodotus and later Aristarchus as pioneers of Alexandrian School of classical philology, devised the instrument of principle of analogy aimed at interpreting and authenticating Homeric and Hesiodic canons. Its exact reconstruction is still difficult to interpret (Peck, 1911, pp. 98-117).\(^8\) But it roughly amounts to determining the authenticity of the text (or a historical authorship of the text) as the proposed relationship that has to be presupposed between a stream of authorial intentions behind every utterance which is presumed to follow from the regularity of language within the texts of multiple manuscripts. Here, the X’s are the textual utterances (verses of poems for instance), R is the regularity of language between them, Y’s are the corresponding authorial intentions behind every X, and thus the avatar of this R is the unity of consciousness which is sought as the historical authorship. This helped build canons (manuals to exemplify the excellence of poetic geniuses) from Alexandrian School for Homer’s Iliad and Odyssey on one hand, and Hesiod’s Theogony on the other (Peck, 1911, p. 99ff). Alexandrian philology became the taxonomy, i.e., the classifying scheme of literary entities and phenomena and theoretical physics becomes the taxonomy of physical beings or phenomena. One uses analogy as linguistic regularity, the other uses analogy as Pythagoreanism. The parallelism is significant. Neither justifies itself on this ultimate point of the very use of analogy. The epistemological foundation of theoretical physics as descriptive applicability of pure mathematics lack a sound foundation, similarly classical philology as a theory to resolve conflict of interpretation and historical authorship or authenticity of text requires a justification for the use of analogy-anomaly as a principle of interpretation.\(^9\) This principle of analogy (and anomaly) was then appropriated (with critical revision) by Dilthey in his later hermeneutic turn that regarded human interpretation (Auslegung) as a rule driven aspect of human understanding (Verstehen) as the most fundamental human capacity responsible for the intelligibility of the historical world (Dilthey, 1977).

3. Towards an Unusual Parallelism: Mathematics vs. Subject-Matter of Universal Hermeneutics

Thus the ultimate end product of both (i) classical philology as regional hermeneutics via methodological principle of analogy and (ii) Pythagoreanism as anthropocentrism via its own formalist and non-formalist analogies is the same. It is the understanding embedded in classificatory scheme of taxonomy. Taxonomy is the very nexus by virtue of which an entity is named and thus comes-to-be in language.\(^10\) Taxonomy is the very expression of understanding. Every science is such taxonomy. It is thoroughly linguistic and thus confirms both phenomenological and hermeneutic thesis for the linguistic nature (linguisticity) of human understanding (Verstehen). However, the nature of this linguisticity is different in natural and human sciences.\(^11\) Former is dominated by mathematics as language with theoretical physics as a paradigmatic example. Latter is dominated by restricted-natural language.\(^12\) When philology is taken as a particular human science, then this is dominated by the literariness of language. This includes not only literature and all classical texts as its subject matter but also the human competence which renders it possible, i.e., the subject matter of both rhetoric and poetics (Verburg, 1998, p. 17ff). So when it comes to search for a logical classifying scheme that can unify the two corresponding modes of human understanding, natural vs. human sciences, we have at least two possibilities; it would be either mathematics or the human literary competence that shows itself in regional hermeneutics through literary historical texts.

However, before I move any further, I must answer a possible objection. Theoretical physics is indeed a paradigmatic example for dominance of mathematics as what contributes in rendering the taxonomy of natural beings possible. But philology is not so as-such in human historical world. What I am required is to work out a proper paradigmatic example from the domain of human sciences. To this end, I would recall the classical debate of


Eklären vs. Verstehen (Explanation vs. Understanding) from German thought of 19th century (see (Wright, 1971) for detail).

The technical distinction between Eklären and Verstehen was first introduced by J. G. Droysen in his Grundriss der Historik (1858) which was later adopted and extended by Wilhelm Dilthey through his Einleitung in die Geisteswissenschaften (1883). If we can avoid subtleties then both Droysen and Dilthey’s use of the term was primarily motivated by the thought of practicing philologist and Protestant theologian Friedrich Schleiermacher, in particular, his universal hermeneutics, to the extent that this use of the term coincided in at least one fundamental point: natural sciences has the claim to epistemological autonomy in the sphere of nature with their own distinguished methodology and human sciences (Geisteswissenschaften) have their own claim to epistemological autonomy in the sphere of the ‘human’ with their distinguished methodology. Eklären was termed as the methodology for the former whereas Verstehen was so termed for the latter (Apel, 1982). The model thought for Eklären could be considered as expounded in Kant’s first critique, i.e. the Critique of Pure Reason. Against this, Dilthey in particular sought to lay down the foundations for Verstehen through his grand project of Critique of Historical Reason (Dilthey, 2002). Dilthey’s pursuit of this objective eventually found its more comprehensive results in his later hermeneutic works, more significantly The Rise of Hermeneutics (1900) and the Understanding of Other Persons and Their Expressions of Life (1910). It is in the latter where he explicitly works out how Verstehen actually works through his hermeneutical reflections motivated by the works of Schleiermacher.14 There one can see how he explicitly works out the graded process of Verstehen with strong philological underpinnings thereby rendering hermeneutics as the theory of Verstehen. On the other hand in (Dilthey, The Rise of Hermeneutics, 1996b) he explicitly elaborates how hermeneutics is the science of the individual person (in the most comprehensive and extended sense) and thereby establishes hermeneutics as what explicates the methodology for human sciences. Thus, it is from classical philology, through the works of Schleiermacher, that Dilthey works out a universal nexus or taxonomy for the human historical world, since what is responsible for explicating Verstehen should exactly be considered responsible for determining the sense and meanings in all forms of human actions. This universal taxonomy is Dilthey’s hermeneutics so grounded (the seeds of which Dilthey explicitly acknowledged in Schleiermacher’s universal hermeneutics, see (Dilthey, 1996a, p. 132). As a human scientific discipline, this universality of hermeneutics makes it also the paradigm example of human sciences.

I had worked out the correlation between (i) how mathematics was dominating the taxonomical significance of natural sciences, and (ii) how philology was the taxonomy for the historical world in context of literary text. Once latter is seen connected with Dilthey’s hermeneutics as a universal taxonomy for the human historical world, we can draw a possibility of extending the correlation to (iii) how history through historical texts as models of human actions dominates hermeneutics as universal taxonomy of human or historical sphere.15 This would help see the role of historical texts as playing the same role in hermeneutics that mathematics plays in the natural sciences in general and theoretical physics in particular. The difference is that mathematics, totally unlike ‘historical texts’, is recognized as a well-defined scientific discipline. On contrary, ‘historical texts’ only defines the primary subject-matter of hermeneutics as human or historical scientific discipline but itself does not constitute a scientific discipline in its own right. So the correlational analogy seems to have an odd term in it. But the oddness is only apparent and superficial, for what constitutes the very discipline of mathematics is not some logical axiomatic foundation from which the whole mathematics could be derived as logical inference. This was classical logicist dream pursued at the turn of twentieth century in the project of Principia Mathematica pioneered by A. N. Whitehead and Bertrand Russell. But the project failed once Kurt Gödel’s Incompleteness Theorems are acknowledged, according to which any formal theory sufficiently complete to include Arithmetic could not prove every theorem stated within the very formal theory itself (Smornsyki, 1977). Thus mathematics is not a logically founded scientific discipline with single set of axioms and rules of logical inferences. Without recourse to the institutional practices of mathematics, one can’t take a cognitive stance over the meanings of mathematics. This does not mean the triviality that ‘mathematics is what mathematicians do’, but only the insufficiency of a foundationalist sort of epistemology. This is also evident in how Saunders Mac Lane16 identifies the characterizing properties of mathematics. According to him, these are conundrums, completion, invariance, analogy, intrinsic structures... axiomatizations, etc. the list is quite long (Lane, 1986, p. 36ff). These are merely, what Ludwig Wittgenstein may have called, the family resemblances within the language-games involving mathematics. Thus it is not only Steiner’s account of mathematics as anthropocentrism with applicability of mathematics guided by his account of Pythagorean analogies, which is anti-foundationalist, Mac Lane’s account of the very practice of mathematics too shows that there is no singular definition or essential conceptualizing that can characterize the activity of mathematics as a particular scientific discipline. Mathematics has acquired such a status of an apparent scientific disciplinary unity because of its long historical significance. In particular, its role for inspiring epistemological models of philosophizing which started with Plato, when in his dialogue Republic, he placed the value of mathematical knowledge only next to the dialectics as the characteristic way of philosophizing.17

Thus it is not the oddness that needs to be resolved in order to fix the argument of this paper’s line of investigation. This oddness is not a conceptual weakness but a historical indicator of our being too accustomed to view mathematics as an essentially defined scientific discipline. On contrary, it is institutionally and historically determined. A counterpart of such a discipline in case of (iii) above would be the one which sets historical texts as its subject matter or it would be the discipline that would describe the human competence for historical textualizing. This would nothing other than universal hermeneutics.
4. Concluding Remarks and Suggestions with Pedagogical Ends

Our conceptions of mathematics matter when it comes to mathematical pedagogy, whether we are concerned with designing or improving curriculum, or trying to address teaching related methodological issues. It is customary treat mathematics as a body of established knowledge with sound and certain epistemological foundations of logicist, formalist or Platonist type. This gives the impression that one just has to master the method and conceptual apparatus to the extent that mathematics becomes predominantly a mechanical endeavor, which starts with logical definitions followed by examples and related theorems. The very organization of undergraduate and graduate university education of mathematics bears witness to this.\(^\text{18}\)

The investigations pursued in this paper develop an alternative perspective on viewing what mathematics is all about. This is constituted by hermeneutical reflections to the extent that the author has mainly tried to work out the interpretive framework under which mathematics can make sense from the angle of applicability and not from abstract philosophical theorizing. The interpretive framework is the very parallelism between Pythagoreanism as anthropocentrism and practices of regional hermeneutics towards an all-encompassing universal hermeneutics. I propose that a stronger work in this direction is possible. Instead of just parallelism, a hermeneutic-phenomenological correlation is possible. But this would involve a deconstructive historical ontology of Heideggerian type of the historical significance of mathematics, i.e. a hermeneutic-phenomenological examination of the role of mathematics in all human expressions, philosophy, sciences, culture and art throughout in history This is a huge task and is thus beyond the scope of this paper.

Bibliography


Dilthey, W. (1989). Metaphysics as the Foundation of Human Sciences: In Wilhelm Dilthey's Selected Works: An Introduction to Human Sciences, culture and art throughout in history. This is a huge task and is thus beyond the scope of this paper.


Notes

1 This use of the term Platonism encompasses (although inclusively) its classical origin in Plato’s metaphysics as the world of forms. This was a particular case of metaphysical realism. Modern versions of Platonisms are all realisms that mostly remain silent about the exact nature of metaphysical existence of such independence. Mostly their point of departure is the pioneering semantic analysis of mathematical concepts and statements as pursued by Frege’s logicist Platonism (Dummett, 1991, pp. 303). From this point onwards, this study will not distinguish between realism and Platonism.

2 Hermeneutics mainly refers to the art of interpretation. Its specialized or regional modifications involve literal, legal, Biblical and philological (Hirsch, 1967, p. vii). Unless prefixed by the word ‘regional’ or specialized, hermeneutics would always be meant in a more philosophical sense as pioneered by Friedrich Schleiermacher and Wilhelm Dilthey.

3 Former is a logicist-Platonist (Steiner, 1998, p. 21f) in context of the semantical analysis of mathematical objects and relations which he finds perfectly in accord with Fregean semantics, whereas the latter is a nominalist with formalist bent. Also, Steiner’s Platonism is a kind of restricted Platonism (thus we have the qualification: logicist-Platonism) in the sense that according to him, Fregean semantical analysis of mathematical concepts is restricted to the application of mathematical concepts in general as second-order predicative relations the extension of which includes physical concepts as first-order predicative relations. The focus of Fregean semantic analysis was never the application of mathematical concepts to natural objects but only the logical validity by virtue of which second-order reasoning become applicable to first-order reasoning. Thus as Frege himself regarded mathematical object in general as “they are the laws of the laws of nature”. Steiner on the other hand is concerned with how mathematical concepts together apply to nature itself in context of lawfully describing the physical phenomena and how they become predictable (Steiner, 1998, p. 25).

4 The suspension of judgment with any kind of metaphysical positing has been recognized primarily by phenomenology as a legitimate starting for any radical investigation. Usually Dilthey’s hermeneutics is not considered to be correlated with phenomenology, especially when he started to view Husserlian transcendental phenomenology more criti-
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cally. This is certainly not the place to work out how much phenomenology is a presupposition for Dilthey’s hermeneutics. However, if I not difficult to see how phenomenological reduction which first and foremost aims at overcoming metaphysical positing as a result of ‘epoché’ which is a phenomenological consequence of adhering to the phenomenological maxim “back to things themselves” (Husserl, 2001, p. 88), and making Husserl’s principle of all principles (Husserl, 1983, p. 44) as its point of departure, logically correlates with Dilthey’s own principle of all principles that renders consciousness as what determines beings-as-beings (Dilthey, 2002, p. 247).

Pythagoreanism amounts to the doctrine that the natural kinds are exactly what are determined by mathematical kinds. Steiner worked out many such formalist and non-formalist mathematical analogies to exemplify his concept at different nth-order. I am omitting the general strategy and examples of them as they require most advanced understanding of both theoretical physics and geometric methods which is beyond the scope of this paper. See chapter 4 for a detailed construction of them.

In order to avoid confusion with the concept of ‘analogy’, I would be using the word parallel or parallelism.

This is a simplified version of strategy (1) of 1st order non-formalist Pythagorean analogy. There are at least 6 general strategies for both formalist and non-formalist type.

See also (Myres, 2014, p. 30f) and (Dilthey, The Rise of Hermeneutics, 1996b) for this principle and its role in determining canonical texts from classics. Principle of analogy has been used extensively, although with varying degree of conceptualizing; throughout in the practice of philology, see (Verburg, 1998) for detail, which also includes the Alexandrian School.

Steiner had to resort to the aesthetics of anthropocentrism as the ultimate basis instead of a sound epistemology (Steiner, 1998, p. 55ff).

For instance, existence of Omega minus particle was posited merely as a result of mathematical symmetry that existed between SU(2) to SU(3). This was immediately believed to be existent, and later was indeed found to be existent! This is not just one instance! There are numerous such instances in which an object is claimed merely by virtue of some analogy from mathematics and is later found to be existent. There is no sufficient justification for all this. But it shows how theoretical physics is such a taxonomy or the nexus by virtue of which things comes to be in language. See (Steiner, 1998) for more examples. On the other hand, classical philology brought classical texts (like Homer’s Iliad and Odyssey, Hesiod’s Theogony) along with the historical Homer and Hesiod whose historical existence as single author was always pretty much contestable, see (Myres, 2014, p. 30)

I am using this phrase ‘human sciences’ as the translation of Dilthey’s term Geisteswissenschaften which encompasses the disciplinary domains of both Arts & Humanities (Performing Arts, Literary Studies, Linguistics, etc.) and Social Sciences (Sociology, Political Sciences, Anthropology, etc.) as generally held in modern universities. So, whenever this term is used throughout in this paper, its all-inclusive meanings should be taken. See the trans. /ed. note in (Dilthey, The Imagination of the Poet: Elements for a Poetics, 1985, p. 36n)

Restricted in the sense that the discourse of human science after all involves specialized vocabulary that modifies natural language into a particular language game.

Hermeneutics for Dilthey (and Schleiermacher as well) is always concerned with interpreting written monuments. Thus it was first seen as generalizing regional hermeneutics (Dilthey, The Rise of Hermeneutics, 1996b)

See (Dilthey, The Understanding of Other Persons and Their Expressions of Life, 1977) for the detailed interpretive account of Verstehen and (Dilthey, Schleiermacher’s Hermeneutical System in Relation to Earlier Protestant Hermeneutics, 1996a) for how much Dilthey himself owes to Schleiermacher.

I am fully consciousness of a threat of logical circularity in this last statement. But the circularity is only apparent as far as its logical sense is concerned, for it is hermeneutic circularity that exhibits here in the repetition of ‘history’ and the ‘historical’. Former is part and the latter is the whole. Former is determined by historical texts as historical human actions through which latter as the all-encompassing nexus of human existence shows itself. See the detail of hermeneutic circle in (Scholtz, 2015, p. 65)

20th century renowned mathematician and a significant contributor to the discipline of both algebraic topology and Homological algebra in pure mathematics along with Cartan Eilenberg and Alexander Steenrood

Cf. Plato, Republic, 1997, pp. 1129-1131 (509d-511c)) for divided-line analogy, also, see (Plato, Republic, 1997, pp. 1132-1137 (512a-520a)) and (Plato, 1997, p. 484-494 (201d-212c)) for how Plato makes sense of the intellectual journey towards the encounter with reality allegorically which correlates with former.

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