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Cheap Talk with Outside Options

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Abstract. In Crawford and Sobel (1982) (CS), a sender (S) uses cheap talk to persuade a receiver (R) to select an action as profitable to S as possible. This paper shows that the presence of an outside option – that is, allowing R to avoid taking any action, yielding state-independent reservation utilities to R and S – has an important qualitative impact on the results. Contrary to CS, in this model, the informativeness of communication is not always decreasing in the level of conflict of interest. Relatedly, communication can be more informative than in CS.

Keywords: Cheap Talk, Information Transmission, Experts.

JEL Classification Numbers: D82, D83.

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1 Introduction

In this paper, we add an element to the celebrated Crawford and Sobel (1982) model of cheap talk, hereafter CS. We assume that the receiver (R) may respond to the message of the sender (S) by taking no action. This "inaction," interpreted as an outside option, yields each player a state-independent reservation utility. Surprisingly, we find that the results of this model are quite different from those of CS. An important insight of CS is that as the level of conflict of interest between R and S increases, the informativeness of communication decreases; this is not true in our model. In this model, communication may become more informative as the conflict of interest between R and S increases.

It is not difficult to imagine scenarios where R may opt out from the relationship with S and avoid taking any action. Consider, for instance, the case of an expert in an organization who makes a proposal to senior management on an important issue of business strategy. Clearly, the senior management can implement a modified version of the proposal, accept the proposal, or ignore the proposal and continue with business as usual. Consumers' investment decisions also apply. For example, consider a bank and its private customer. The customer has some amount of money in his checking account. The bank recommends its investment product to the customer. The customer may be persuaded and decide how much he will invest in it. Alternatively, the customer may decide to not invest in it at all and leave his money in his checking account because he thinks he does not have sufficient information to take the risk. Returns from "action" of investments are uncertain and dependent on the state of the financial market while returns from the "inaction" of not investing are predetermined and independent of the state.¹

Thus, the assumption in CS that R must take an action is not always realistic. Of course, if the assumption does not have a significant impact on the results, the simplification poses no problems. But this paper shows otherwise. CS predicts that, when conflicts of interest are large, little information is transmitted. Thus, in organizations, management should always employ people whose biases are as low as possible.² However, this conclusion is not valid in this model.

To see the intuition for the result that conflict of interest can facilitate communic-

¹As a practical matter, the "inaction" of not investing is different from the "action" of investing zero if, for example, there are transactions costs associated with having an investment account.

²See Crawford and Sobel (1982) and Dessein (2002).

ation, consider the problem of a project manager in a firm.³ The firm is considering starting a new product line. The manager will decide the scale of a new investment as in CS. But in this model the manager has an additional option: he can choose to continue with business as usual and make no investment. The consequences of business as usual are predictable to everyone in the firm. But the consequences of new investments depend on technical issues which are only known to an expert in the firm. The firm's profit is maximized if the manager fully internalizes the expert's information and selects the optimal investment. Hence, the manager asks the expert for advice about the new investment before making a decision. Increasing the firm's profit is beneficial for both agents, but the expert is biased toward a larger investment than the manager by some fixed amount b in order to benefit his future career. Hence, in this example, the manager is R , the expert is S , the scale of the new investment represents an action, business as usual represents inaction, and b represents conflict of interest.

The biased expert does not report the firm's optimal investment to the manager exactly; he partitions his information into finitely many categories. In addition, as in CS, his advice gets noisier as the firm's optimal investment gets larger. For example, the expert may use one of three categories, "small," "medium" and "large," where "large" is the coarsest category. On the other hand, the manager has single peaked preferences conditional on the truth and hence selects an investment only if he has a lot of information about a new investment. Hence, the manager selects business as usual when the expert reports "large" because the information is too noisy, that is, lacking details of information that the manager needs to confidently make an investment. He selects a new investment when the expert reports "small" or "medium."

In this case, when the expert finds the firm's optimal investment is large, he may have an incentive to lie and report "medium." Let us compare a more biased expert (i.e., b is large) and a less biased expert (i.e., b is small). First, suppose the more biased expert finds the firm's optimal investment is large. He honestly reports "large" so that the manager selects business as usual because an investment which he can induce by reporting "medium" is far from what the expert prefers because the conflict of interest is large. Next, suppose the less biased expert finds the firm's optimal investment is large. He now has a great incentive to lie and report "medium" because an investment he can induce by reporting "medium" is close to what

³We thank Andrew Newman for suggesting this example.

he prefers. The less biased expert can not use three categories honestly and hence conveys less information. In other words, the more biased expert can use a larger number of categories in his report because he has a smaller incentive for reporting "medium." Hence, conflict of interest can facilitate communication.

The remainder of the paper is organized as follows: Section 2 reviews related literature. Section 3 describes the basic model. Section 4 provides a characterization of equilibria. Section 5 deals with a welfare analysis. Section 6 explains comparative statics. Section 7 compares outcomes in this model and CS. Section 8 concludes the paper. Proofs are in the appendix.

2 Related Literature

This paper is most closely related to CS's analysis of information transmission as mentioned in Section 1. Our study is also related to the economics of pandering: S biases his advice toward actions that look better for R given R's prior belief. Che, Dessein and Kartik (2013) study the economics of pandering using a cheap talk model with an outside option but with a different focus from this paper. Che, Dessein and Kartik mainly consider the case where R and S have identical preferences over actions and so fix the level of conflict of interest.⁴ They assume that the payoffs from the outside option can be different between R and S. Thus, if the outside option is relatively profitable for R but not for S, S biases his recommendation toward an action that ex-ante looks better for R in order to prevent R from selecting the outside option, even when S knows some other action makes both agents better off. The presence of the outside option can worsen information transmission.⁵ On the other hand, this paper compares different levels of conflict of interest and shows that information transmission can be non-monotonic in the conflict of interest in the presence of the outside option.

Caillaud and Tirole (2007) study economics of pandering by comparing different

⁴They also consider an extension where R and S have different preferences over actions in their supplementary appendix.

⁵Brandenburger and Polak (1996) and Maskin and Tirole (2004) also show that S's incentive for pandering can worsen information being transmitted to R. Brandenburger and Polak show that when a corporate manager cares about his firm's short-run stock price, he will distort his investment decision toward what, the market believes, is the best even if he knows it is not the best for the firm's long-run profit. Maskin and Tirole show that in representative democracy, politicians may pander to public opinion and select a suboptimal popular policy rather than the right policy for society so that they will get reelected. In these models, it is actions that are distorted by an informed agent, not communication directly.

levels of conflict of interest but consider a different situation from this paper. In Caillaud and Tirole, S persuades R to accept his proposal by sending a report. S does not have any information relevant to R and hence there is no information transmission per se. R can verify the report at his cost before making a decision or ignore the report. On the other hand, in this paper S sends a cheap talk message to R and persuades R to select a project as profitable to S as possible. Nevertheless, Caillaud and Tirole come to a similar conclusion to this paper that R's information acquisition decision can be non-monotonic in the conflict of interest.⁶

Many studies have used games of communication to understand issues in organizational economics. Landier, Sraer and Thesmar (2009) and this paper show that larger conflicts of interest can allow the organization to utilize local information more effectively. However, the approaches are different between the two papers. Landier, Sraer and Thesmar study a costly signaling model where there is an informed manager (S) and an uninformed worker (R) in the firm. The manager selects a project. But for the project to succeed, the worker should implement it by putting in his best efforts. Thus, the manager may intentionally select the suboptimal project for the firm in order to motivate the worker. On the other hand, this paper studies a costless signaling model – a cheap talk model – where there is an uninformed manager (R) and an informed expert (S). The manager selects and implements a project. The expert does not fully reveal his information to the manager in order to persuade the manager to select and start a project as profitable to the expert as possible.

Chiba and Leong (2013) study managerial issues by using a cheap talk model with an outside option, but have a different focus from this paper. Chiba and Leong use a simple model where there are two states and two actions and study practical managerial issues such as interpersonal authority and transparency about S's bias.⁷

⁶In Caillaud and Tirole, S's benefit from the proposed project is positive, which is common knowledge. But R's benefit can be positive or negative, which is apriori unknown to anyone. R can know her benefit from the project only by verifying a report given by S. The conflict of interest is measured by R's prior that the project is unsuccessful for R. Caillaud and Tirole show that R verifies the report sent by S only when the conflict of interest is medium. R ignores the report and denies the proposal when the conflict of interest is large. R accepts the proposal without verifying the report when the conflict of interest is small.

⁷In Chiba and Leong (2013), there are only two states and two actions. Again, R has an outside option to take inaction. Also, an action is costly for R and inaction is costless for both players. Hence, S may recommend a suboptimal action which is ex-ante preferred by R in order to convince R to take an action. If S has more similar preferences to R, he may have a larger incentive to distort his message. Chiba and Leong explore interesting managerial issues. The first issue is related to the manager (S)'s choice between interpersonal authority and communication to make the worker (R) obey. The second is related to how transparency about the bias of a financial adviser (S) affects the quality of his advice

In contrast to Chiba and Leong, this paper directly extends CS where there are continuum states and continuum actions and focuses on fundamental issues such as the relationship between conflicts of interest and information transmission.

Matthews (1989) studied a bargaining game including a veto stage and introduced outside options in alternative ways in cheap talk models. In his game, S instead of R has an option to selection inaction. More specifically, S can either accept R's proposal of an action or veto the proposal and select inaction. In Matthews, unlike in our model and CS, R's ideal action is independent of S's information, fixed and commonly known, while S's ideal action depends on his private information. To prevent S from vetoing a proposal, R has an incentive to pander and propose an action which seems to R closer to S's favorite outcome.⁸

3 Model

There are two players, the receiver (R) and the sender (S). R chooses either some action $a \in \mathbb{R}$ or the outside option of inaction $a = \emptyset$. There is a state of the world, denoted θ . Nature chooses θ according to a uniform distribution with support $[0,1]$. If R selects an action, its consequences depend on the state. On the other hand, if R selects the outside option of inaction, its consequences are already known to everyone. Only S observes the true realization of θ . S reports the state of the world using a cheap talk message $m \in [0,1]$ to R, after which R chooses a .

Let U^j denote the payoff function of player $j=R, S$. When some action $a \in \mathbb{R}$ is selected, each player's payoff is given by a standard quadratic loss function:

$$\begin{aligned} U^R &= -(a - \theta)^2, \\ U^S &= -(a - \theta - b)^2, \end{aligned} \tag{1}$$

where $b > 0$. When the outside option of inaction $a = \emptyset$ is selected, each player's

toward his client (R). The third is related to the expert (S)'s costly effort to acquire information before he gives advice to the manager (R).

⁸Shimizu (2012) also considers a model including an exit stage where S has an outside option to exit. In Shimizu, like in our model and CS, both players' ideal actions depend on S's information. Conflict of interest over ideal actions can facilitate information transmission because R may have an incentive to select an action which seems to R closer to S's favorite action, which may encourage S to reveal his information.

payoff is given by a state independent reservation utility:

$$\begin{aligned} U^R &= -u_R, \\ U^S &= -u_S, \end{aligned} \tag{2}$$

where $u_R > 0$ and $u_S > 0$. All aspects of the game except θ are common knowledge.

The timeline is as follows. First, nature chooses the state θ according to a uniform distribution with support $[0,1]$, which is observed by S. Second, S reports the state using a cheap talk message $m \in [0,1]$ to R. Third, R decides whether to take some action, denoted $a \in \mathbb{R}$, or inaction, denoted $a = \emptyset$. Finally, the outcome is realized for every player.

Given any θ , R's ideal decision is $a = \theta$, and S's ideal decision is $a = \theta + b$. Thus, b measures the level of conflict of interest as in CS.

This model is almost equivalent to CS. The presence of the outside option of inaction is the only difference from CS.

4 Equilibria

This section provides a full characterization of the equilibria of this model.

The solution concept is Perfect Bayesian equilibrium (PBE). As in CS, every equilibrium in this model is characterized by a finite partition.

A strategy for S is a function $m(\theta)$ that associates each state θ with the message $m(\theta)$ used by S in that state. A strategy for R is a function $a(m)$ that associates each message m with the decision $a(m)$ selected by R given that message. A belief for R is a function $\mu(\theta|m)$, where $\mu(\theta|m) \geq 0$ and $\int_{\theta \in [0,1]} \mu(\theta|m) = 1$, that gives R's posterior as a function of m by Bayes rule.

First, we are going to show that every equilibrium is partitional. We let $\theta(N) \equiv (\theta_0(N), \dots, \theta_N(N))$ denote a partition of $[0,1]$ with N steps, where $0 = \theta_0(N) < \theta_1(N) < \dots < \theta_N(N) = 1$. We let $(\theta_{i-1}(N), \theta_i(N))$ denote the i -th step of partition $\theta(N)$.

A *partition equilibrium with N steps* is an equilibrium in which S partitions his information into N steps, denoted by $\theta(N)$, and only reveals which step of $\theta(N)$ his information lies in.⁹ For example, suppose a partition equilibrium with N steps

⁹Precisely, a partition equilibrium in this paper is a *monotone partition equilibrium* where S does not mix among messages that imply different steps unless that the state is at the boundary point between

is played, and the state is $\theta \in (\theta_{i-1}(N), \theta_i(N))$. S only reveals that the state is in the i -th step. Given the information revealed by S, R updates his belief about the state θ using Bayes rule, and selects his optimal action $a \in \mathbb{R}$ or the outside option of inaction $a = \emptyset$.

Lemma 1 *Consider an equilibrium where R selects an action on the equilibrium path. Every equilibrium is a partition equilibrium with a finite number of steps.*

Proof. See Appendix. ■

Lemma 1 states that in this model, every equilibrium where at least one action is induced consists of a finite partition. Similar to CS, in such an equilibrium, a finite number of actions are induced. Because of conflict of interest, truthfully revealing the exact state is not an equilibrium.

Two types of partition equilibria may exist in this model. The first type of partition equilibria are ones where an action is always taken ("ACTs"). The second type of partition equilibria are ones where sometimes the outside option of inaction is chosen ("STOPs"). Every ACT is an equilibrium in CS because inaction is not an option for R in CS.

To ensure that there are no off-equilibrium messages, we will follow an approach used by CS and focus on equilibria where S randomizes as follows. Consider a partition equilibrium with N steps. Given the state $\theta \in (\theta_{i-1}(N), \theta_i(N))$, S sends a message m that is randomly drawn from $(\theta_{i-1}(N), \theta_i(N))$.¹⁰

In addition, when there is a STOP with one step (a babbling STOP), there may also exist a STOP with more than one step such that R always selects inaction given any message, and hence each player's payoff is equivalent to that in a babbling STOP for every state. In this case, we focus on a STOP with one step.¹¹

In any type of equilibrium, when R selects an action in the i -th step, as in CS, his action must be given by:

$$a(m) = \frac{\theta_{i-1}(N) + \theta_i(N)}{2} \quad \text{for } m \in (\theta_{i-1}(N), \theta_i(N)) \quad \text{for } i \in \{1, \dots, N\}. \quad (3)$$

two neighboring steps. In this paper, when we say a partition equilibrium, we are referring to a monotone partition equilibrium.

¹⁰Every m only reveals that the state is in the i -th step of partition $\theta(N)$. As CS's Theorem 1 shows, all equilibria given by partition $\theta(N)$ are essentially equivalent to those with this uniform signaling rule.

¹¹We thank Navin Kartik for pointing out this issue.

R's ideal decision is action $a = \theta$ if R knows the state θ . In a partition equilibrium R selects the average of his ideal actions across states in the step, which is denoted by (3), because R only knows that the state is in the i -th step.

In an ACT with N steps, it is easy to see that the partition must satisfy the same indifference conditions for S as in CS:

$$-\left(\frac{\theta_{i-1}(N)+\theta_i(N)}{2}-\theta_i(N)-b\right)^2 = -\left(\frac{\theta_i(N)+\theta_{i+1}(N)}{2}-\theta_i(N)-b\right)^2 \quad (4)$$

for $i \in \{1, \dots, N-1\}$.

(4) is simplified to:

$$\theta_{i+1}(N) - \theta_i(N) = \theta_i(N) - \theta_{i-1}(N) + 4b \text{ for } i \in \{1, \dots, N-1\}. \quad (5)$$

Finally, these equilibria differ from equilibria in CS only in that we must ensure that R does not choose inaction, so we require optimality conditions for R:

$$\frac{-(\theta_i(N)-\theta_{i-1}(N))^2}{12} \geq -u_R \text{ for } i \in \{1, \dots, N\}. \quad (6)$$

(6) is simplified to:

$$\theta_i(N) - \theta_{i-1}(N) \leq \sqrt{12u_R} \text{ for } i \in \{1, \dots, N\}. \quad (7)$$

Equation (5) means that in an ACT, given b , the higher the state, the wider the step, and the less precise the information.¹² Inequality (7), meaning that R has sufficient information about the state, is necessary for R to always select an action because R's payoff function has a single peak $a = \theta$. Each step should be narrow enough that R's expected payoff from his optimal action exceeds his payoff from inaction. Figure 1 graphically shows S's indifference conditions and R's optimality conditions in an ACT with $N = 4$ steps.

¹²Since S prefers a higher action than what R actually wants, S has an incentive to declare a higher state than the true state. In order for S to tell the truth, any rightward step of the partition must be wider than its adjacent leftward step. Suppose the state is at the boundary point of the partition between two adjacent steps. S has an incentive to declare he is in the rightward step. In order to prevent S from doing this, R must take a higher action in the rightward step than S wants. R takes a higher action in the rightward step as the rightward step becomes wider. The rightward step must be wider than the leftward step, and should be wider as S is more biased or b increases.

Figure 1: ACT with N=4 steps

In an ACT, given b , the higher the state, the wider the step. On the other hand, R's payoff function has a single peak $a = \theta$ and depends on the distance between an action and the state $|a - \theta|$; R prefers the outside option of inaction to any action if and only if the reported step is sufficiently wide. Hence, the higher the state, the more R's willingness to select inaction.¹³ Hence, if R selects inaction, he will do so in the N -th step.¹⁴

Lemma 2 *Consider a STOP with N steps such that an action is selected on the equilibrium path where $N \geq 2$. Inaction is induced only if the state belongs to the N -th step.*

Proof. See Appendix. ■

In a STOP with N steps where $N \geq 2$, R will select inaction when S reveals "the state is in the N -th step" and an action given by (3) otherwise:

$$a(m) = \begin{cases} \frac{\theta_{i-1}(N) + \theta_i(N)}{2} & \text{for } m \in (\theta_{i-1}(N), \theta_i(N)) \text{ for } i \in \{1, \dots, N-1\} \\ \emptyset & \text{for } m \in (\theta_{N-1}(N), 1). \end{cases} \quad (8)$$

Consequently, S's indifference condition changes at the $(N-1)$ -th boundary point, $\theta_{N-1}(N)$. At $\theta_{N-1}(N)$, S should be indifferent between R's action in the $(N-1)$ -th step and the outside option of inaction selected in the N -th step. The indifference

¹³It is not hard to extend this result to the case where R's utility function is any decreasing function of $|a - \theta|$.

¹⁴It is not hard to extend this result to the case where S's utility function is any decreasing function of $|a - \theta - b|$. Because of the symmetric property, there does not exist a STOP where an action is induced in the N -th step.

condition for S at boundary point $\theta_{N-1}(N)$ changes to:

$$-\left(\frac{\theta_{N-2}(N) + \theta_{N-1}(N)}{2} - \theta_{N-1}(N) - b\right)^2 = -u_S. \quad (9)$$

(9) is simplified to:

$$\theta_{N-1}(N) - \theta_{N-2}(N) = 2\sqrt{u_S} - 2b. \quad (10)$$

But for a STOP with N steps where $N \geq 3$, S's indifference conditions at the remaining boundary points remain unchanged:

$$\theta_{i+1}(N) - \theta_i(N) = \theta_i(N) - \theta_{i-1}(N) + 4b \quad \text{for } i \in \{1, \dots, N-2\}. \quad (11)$$

R's optimality conditions are now:

$$\begin{aligned} \theta_i(N) - \theta_{i-1}(N) &\leq \sqrt{12u_R} \quad \text{for } i \in \{1, \dots, N-1\}, \\ \theta_i(N) - \theta_{i-1}(N) &\geq \sqrt{12u_R} \quad \text{for } i = N. \end{aligned} \quad (12)$$

Recall $\theta_0(N) = 0$ and $\theta_N(N) = 1$.

Equation (10) means that when u_S is sufficiently small compared to b , there does not exist a STOP with more than one step. In addition, equation (10) determines the width of the $(N-1)$ -th step and hence all of the previous steps, the first step through the $(N-2)$ -th step, as well. Inequalities (12) mean that only the N -th step is wide enough for R to refer the outside option. So, as in an ACT, the higher the state, the wider the step. Figure 2 shows S's indifference conditions and R's optimality conditions in a STOP with $N = 4$ steps.

Figure 2: STOP with N=4 steps

We have explained the conditions defining each type of equilibria. The next question is under what condition does each type of equilibrium exist?

If inaction is unattractive enough for R – specifically, if $u_R > 1/12$ – then the game is equivalent to CS. More specifically, only ACTs exist and the set of ACTs is identical to the set of equilibria in CS for the same parameter values.

Proposition 3 Consider $u_R \in (1/12, \infty)$ and $u_S \in (0, \infty)$. For every b :

(1) There is a positive integer $N^{ACT}(b)$ such that there is an ACT with N steps for every $N \in \{1, 2, \dots, N^{ACT}(b)\}$.

(2) There is no STOP.

Proof. See Appendix. ■

Consider $u_R > 1/12$. From S's indifference condition given by (5), for each $b > 0$, $N^{ACT}(b)$ is defined by:

$$N^{ACT}(b) = \begin{cases} 1 & \text{for } b \in \left[\frac{1}{4}, \infty\right) \\ k & \text{for } b \in \left[\frac{1}{2(k+1)k}, \frac{1}{2k(k-1)}\right), \quad k \geq 2. \end{cases} \quad (13)$$

$N^{ACT}(b)$, monotonically decreasing in b , is equivalent to the maximum number of steps in CS.

For $u_R \leq 1/12$, inaction is sufficiently attractive to R that he may choose it in equilibrium. In this case, an ACT exists only if S reveals enough information. This implies that there is a lower bound for the number of steps for an ACT, in addition to the usual upper bound.

Proposition 4 Consider $u_R \in (0, 1/12]$ and $u_S \in (0, \infty)$. There exists an increasing continuous function $g(u_R)$ and a decreasing step function $L(u_R)$ over $(0, 1/12]$ such that¹⁵:

- (1) For $b \in (0, g(u_R)]$, there is an ACT with N steps for every $N \in \{L(u_R), N^{ACT}(b)\}$.
- (2) For $b \in (g(u_R), \infty)$, there is no ACT.

Proof. See Appendix. ■

Consider $u_R = 1/36$ and any $u_S > 0$. Then, ACT does not exist for $b > g(1/36) \approx 1/20$; an ACT exists otherwise. If an ACT exists, the lower bound of the number of steps is $L(1/36) = 3$.

For $u_R \leq 1/12$, STOP also exists; a STOP with one step (i.e. a babbling STOP) always exists. In addition, multiple STOPS can exist.

Proposition 5 Consider $u_R \in (0, 1/12]$ and $u_S \in (0, \infty)$. For every b , there is a positive integer $N^{STOP}(b, u_R, u_S)$ such that there is a STOP with N steps for every $N \in \{1, 2, \dots, N^{STOP}(b, u_R, u_S)\}$.

Proof. See Appendix. ■

It is not hard to show that there exists a STOP with N steps only if $b \in B(N, u_R, u_S)$, where $N \geq 2$:

$$B(N, u_R, u_S) := \left[\max \left\{ \sqrt{u_S} - \sqrt{3u_R}, \frac{\sqrt{12u_R} + 2(N-1)\sqrt{u_S} - 1}{2(N-1)^2} \right\}, \frac{\sqrt{u_S}}{2^{N-3}} \right) \quad (14)$$

for an integer $N \geq 2$.

So, (14) is derived from S's indifference conditions, given by (10) and (11), and R's optimality conditions, given by (12). For $u_R \leq 1/12$, there always exists a STOP with one step (a babbling STOP). We define $B(1, u_R, u_S) := (0, \infty)$.

Thus, $N^{STOP}(b, u_R, u_S)$ is given by:

$$N^{STOP}(b, u_R, u_S) := \arg \max_N \{N \in \mathbb{Z} : b \in B(N, u_R, u_S)\} \quad (15)$$

In the proof of Proposition 5, we show that $B(N-1, u_R, u_S) \supset B(N, u_R, u_S)$ for any N given u_R and u_S if $B(N, u_R, u_S)$ exists. That is, when a STOP with N' steps exists, a STOP with N steps also exists for every $N \in \{1, 2, \dots, N'\}$.

Propositions 3 through 5 completely characterize the set of equilibria. The next lemma restates the characterization for a fixed conflict of interest b . This characterization is useful for the remaining analysis.

¹⁵The formulas for $g(\cdot)$ and $L(\cdot)$ are given in the Appendix.

Lemma 6 Given b , there are two cutoffs $u_R = C(b)$ and $u_R = 1/12$ such that:¹⁶

- (1) For $u_R \in (0, C(b))$, only STOPs exist.
- (2) For $u_R \in [C(b), 1/12]$, both STOPs and ACTs exist.
- (3) For $u_R \in (1/12, \infty)$, only ACTs exist.

Proof. The result directly follows from Propositions 3 through 5. ■

For $u_R \in [C(b), 1/12]$, there is a lower bound $L(u_R)$ for the number of steps for an ACT; and $N^{STOP}(b, u_R, u_S) \leq N^{ACT}(b)$. For $u_R \in (1/12, \infty)$, the result is equivalent to CS. Figure 3 depicts Lemma 6.

Figure 3: Full Characterization given $b > 0$

For $u_R \leq 1/12$, in addition to STOPs mentioned in this section, there can exist mixed strategy equilibria where R randomizes between taking an action and selecting an outside option in the last step. We will not focus on these mixed strategy equilibria because they are not generic.¹⁷

5 Ex-ante Best Equilibrium in This Model

Section 4 shows the existence of multiple equilibria in this model. This section explains which equilibrium is the best for each player based on his ex-ante expected payoffs.

¹⁶ $C(b)$ is increasing in b and $C(b) \in (0, 1/12)$. The formula for $C(\cdot)$ is given in the Appendix.

¹⁷For example, S partitions the state space into N steps. If S announces that θ belongs to the N -th step, R randomizes between an action $(\theta_{N-1}(N) + 1)/2$ and the outside option with probabilities α and $1 - \alpha$. Otherwise, R selects an action $(\theta_{i-1}(N) + \theta_i(N))/2$. R's optimality condition requires $\sqrt{12}u_R = 1 - \theta_{N-1}(N)$ since R randomizes in the N -th step. S's indifference condition at $\theta_{N-1}(N)$ requires $((\theta_{N-1}(N) - \theta_{N-2}(N))/2 + b)^2 = \alpha((1 - \theta_{N-1}(N))/2 - b)^2 + (1 - \alpha)u_S$. Thus, given b , there can exist a mixed strategy equilibrium with N steps for at most a single (u_R, u_S) pair.

In CS, when there are multiple equilibria, both R and S strictly prefer equilibrium information partitions with more steps. (See CS's Theorem 3 and Theorem 5.) The result in this model differs from the result in CS because there are two types of partition equilibria in this model.

Let $U_j^{STOP}(N)$ denote the ex-ante expected payoffs of player $j=R, S$; and let $\theta_i^{STOP}(N)$ denote the i -th boundary point in an STOP with N steps. So:

$$U_R^{STOP}(N) = \sum_{i=1}^{N-1} \frac{-(\theta_i^{STOP}(N) - \theta_{i-1}^{STOP}(N))^3}{12} - u_R \cdot (\theta_N^{STOP}(N) - \theta_{N-1}^{STOP}(N)), \quad (16)$$

$$U_S^{STOP}(N) = \sum_{i=1}^{N-1} \left(\frac{-(\theta_i^{STOP}(N) - \theta_{i-1}^{STOP}(N))^3}{12} - b^2 \cdot (\theta_i^{STOP}(N) - \theta_{i-1}^{STOP}(N)) \right) - u_S \cdot (\theta_N^{STOP}(N) - \theta_{N-1}^{STOP}(N)), \quad (17)$$

where:

$$\begin{aligned} \theta_i^{STOP}(N) - \theta_{i-1}^{STOP}(N) &= 2\sqrt{u_S} - 2(2N - 1 - 2i) \cdot b \text{ for } i \in \{1, \dots, N-1\}, \\ \theta_N^{STOP}(N) - \theta_{N-1}^{STOP}(N) &= 1 - \sum_{i=1}^{N-1} (\theta_i(N) - \theta_{i-1}(N)). \end{aligned} \quad (18)$$

Let $U_j^{ACT}(N)$ denote the ex-ante expected payoffs of player $j=R, S$; and let $\theta_i^{ACT}(N)$ denote the i -th boundary point in an ACT with N steps. So:

$$U_R^{ACT}(N) = \sum_{i=1}^N \frac{-(\theta_i^{ACT}(N) - \theta_{i-1}^{ACT}(N))^3}{12}, \quad (19)$$

$$U_S^{ACT}(N) = U_R^{ACT}(N) - b^2, \quad (20)$$

where:

$$\theta_i^{ACT}(N) - \theta_{i-1}^{ACT}(N) = \frac{i}{N} - 2(N - i) \cdot i \cdot b \text{ for } i \in \{1, \dots, N-1\}. \quad (21)$$

Holding fixed the type of equilibrium, every player's ex-ante expected payoffs are increasing with the number of steps.

Lemma 7 Fix any $b > 0$, $u_R > 0$ and $u_S > 0$. Then,

(1) Each player $j=R, S$ always strictly prefers an ACT with more steps to an ACT with fewer steps.

(2) Each player $j=R,S$ always strictly prefers a STOP with more steps to a STOP with fewer steps.

Proof. See Appendix. ■

When there are multiple ACTs, the welfare comparison among them is the same as in CS because an ACT is equivalent to an equilibrium in CS. When there are multiple STOPS, the welfare comparison among them is implied by S 's indifference conditions and R 's optimality conditions. S 's indifference conditions (10) and (11) fix the widths of steps where an action is induced. Hence, in order to move from a STOP with N steps to a STOP with $N + 1$ steps, we make the last step narrower and include an acting step as the new first step; and the new first step is preferred by both players to the outside option. Hence, both players prefer a STOP with $N + 1$ to a STOP with N steps.

Within the same type of partition equilibria, an equilibrium with the maximum number of steps is the best for both players. However, as shown in Section 4, given any b , there is a region, given by $u_R \in [C(b), 1/12]$ and $u_S \in (0, \infty)$, where both types of equilibria exist. In this region, we observe $N^{STOP}(b, u_R, u_S) \leq N^{ACT}(b)$, and R is better off in an ACT with $N^{ACT}(b)$ steps than a STOP with $N^{STOP}(b, u_R, u_S)$ steps.

Proposition 8 Fix any $b > 0$, $u_R > 0$ and $u_S > 0$. The best equilibrium for R is an equilibrium with the maximum number of steps. If both types of equilibria have the same number of steps $N^{STOP}(b, u_R, u_S) = N^{ACT}(b)$, the best equilibrium for R is an ACT with $N^{ACT}(b)$ steps.

Proof. See Appendix. ■

Figure 4 depicts the type and the number of steps of the best equilibrium for R .

Figure 4: Best Equilibrium for R given b

Interestingly, the above does not hold for S. Unlike R, more information transmission does not always make S better off. Consider $u_R \in [C(b), 1/12]$ and $u_S \in (0, \infty)$. If u_S is small (i.e., S's benefit from the outside option of inaction, $-u_S$, is large), we observe $N^{STOP}(b, u_R, u_S) < N^{ACT}(b)$. But S is better off in a STOP with $N^{STOP}(b, u_R, u_S)$ steps than in an ACT with $N^{ACT}(b)$ steps.

Proposition 9 Fix any $b > 0$. There exist cutoffs $w(b) \in (0, \infty)$ and $C(b) \in (0, 1/12)$ such that:

(1) If $u_S \in (0, w(b))$ and $u_R \in [C(b), 1/12]$, the best equilibrium for S is a STOP with $N^{STOP}(b, u_R, u_S)$ steps.

(2) Otherwise, the best equilibrium for S is an equilibrium of the maximum number of steps. If both types of equilibria have the same maximum steps, i.e., $N^{STOP}(b, u_R, u_S) = N^{ACT}(b)$, the best equilibrium for S is an ACT with $N^{ACT}(b)$ steps.

Proof. See Appendix. ■

Figure 5 depicts the type and the number of steps of the best equilibrium for S.

Figure 5: Best Equilibrium for S given b

In summary, R always strictly prefers an equilibrium with more steps to an equilibrium with fewer steps, but S may not.

6 Comparative Statics

This section shows that the relationship between the maximum number of steps and the conflict of interest over actions is non-monotonic.

We interpret the number of steps in an equilibrium as a measure of informative-

ness for several reasons.¹⁸ First, we focus on information received by R. As Section 5 shows, R's ex-ante expected payoff is increasing in the number of steps. Hence it seems reasonable to say that more steps is giving R more information.¹⁹

Second, an equilibrium with the maximum number of steps is the "upper bound" in characterizing the set of all equilibria and so is of natural interest.

Therefore, when there are multiple equilibria, we will focus on an equilibrium with the maximum number of steps.²⁰

From now on, we let $N(b, u_R, u_S)$ denote the maximum number of steps in an equilibrium partition in this model, where:

$$N(b, u_R, u_S) = \begin{cases} N^{STOP}(b, u_R, u_S) & \text{if } u_R \in (0, C(b)) \\ \max \{N^{ACT}(b), N^{STOP}(b, u_R, u_S)\} & \text{if } u_R \in [C(b), 1/12] \\ N^{ACT}(b) & \text{if } u_R \in (1/12, \infty). \end{cases} \quad (22)$$

$N(b, u_R, u_S)$ is a measure of the maximum level of information transmission.

We also let $N^{CS}(b)$ denote the number of steps in the most informative partition equilibrium in CS. Recall $N^{CS}(b) = N^{ACT}(b)$, where $N^{ACT}(b)$ is defined by (13) in Section 4.

6.1 Non-Monotonic Relationship between Conflict of Interest and Information Transmission

In the presence of the outside option, the equilibrium structure is determined by R's optimality conditions as well as S's indifference conditions, as shown in Section 4.

¹⁸Our measure of informativeness is different from standard measures such as Blackwell informativeness.

¹⁹In CS, using the number of steps to measure information transmission seems reasonable because both players' ex-ante expected payoffs are increasing in the number of steps.

²⁰A different rationale for focusing on the equilibrium with the maximum number of steps could, in principle, be an equilibrium refinement such as NITS (Chen, Kartik and Sobel 2008) or neologism-proofness (Farrell 1993). Unfortunately, these notions do not isolate the equilibrium with the largest number of steps in this model, even though they do in CS.

Farrell (1993) and Chen, Kartik and Sobel (2008) stated that standard equilibrium refinements for signaling games had no power to refine equilibrium in a cheap talk model because any equilibrium outcome could be supported with an equilibrium where all messages were sent on the path. And, Farrell defined neologism proofness, which selects an equilibrium that admits no-selfsignaling set. As Farrell argued, neologism proof equilibrium does not always exist in CS and hence in this model especially for $u_R < 1/12$. And, Chen, Kartik and Sobel identified NITS that selected an equilibrium where given $\theta = 0$, S weakly prefers the outcome to credibly revealing θ in CS. Unlike in CS, NITS does not always select among equilibria in this model due to the presence of the state independent outside option. Suppose S's payoff from the outside option, u_S , is sufficiently large such that a STOP with two steps exist, but is also sufficiently small that S prefers the outside option to an action b given the state $\theta = 0$. A babbling STOP and a STOP with two steps satisfy NITS.

These two conditions drive the main result that information transmission, as measured by $N(b, u_R, u_S)$, can be non-monotonic in the conflict of interest, b .

In a partition equilibrium, S must be indifferent at each boundary point of the equilibrium partition. Suppose that a partition equilibrium with N steps exists given some level of conflict of interest b . If S observes the state at the i -th boundary point for $i \in \{1, 2, \dots, N-1\}$, S should be indifferent between reporting "the i -th step" and "the $i+1$ -th step." However, if b decreases but the partition remains unchanged, S is not indifferent any more at any boundary point. Hence, if b decreases, the partition should be adjusted so that S is indifferent at every boundary point again. Understanding how the partition is adjusted to restore indifference helps explain the relationship between information transmission $N(b, u_R, u_S)$ and conflict of interest b .

First, consider CS given some b such that $N^{CS}(b) = 3$. S 's indifference conditions decide how the equilibrium partition is adjusted to a decrease in b . For S to be indifferent at each boundary point, the right step should be larger than the left step by $4b$ because S is biased towards a larger action.

Suppose S is less biased (i.e., b decreases). For an equilibrium with three steps to exist, difference between two steps with an adjacent to each other should be smaller; every boundary point shifts right, as shown in Figure 6.

Figure 6: Adjustment in an Equilibrium in CS

Hence, for some $b' (< b)$, we can add a step to the partition (i.e., $N^{CS}(b') = 4$) so that S is indifferent at every boundary point. Therefore, in CS, the larger the conflict of interest, the smaller the maximum number of steps.

The relationship between $N^{CS}(b)$ and b is monotonic as shown in Figure 7.

Figure 7: Monotonicity in CS Model

However, in this model, the result can be different when the outside option of inaction is profitable for R, that is, u_R is small, because R's optimality conditions also affect the equilibrium structure. Consider u_R , u_S and b , for which there is no ACT and the most informative equilibrium is a STOP with three steps so $N(b, u_R, u_S) = 3$.²¹ R's optimality condition requires the width of the last step to be anything wider than $\sqrt{12u_R}$.

Suppose S is less biased (i.e., b decreases). For S to be indifferent at every boundary point in a STOP, as in CS, the boundary point should shift right. For some $b'(< b)$, the last step can be less than $\sqrt{12u_R}$, as shown in Figure 8. R selects an action in the last step, but this makes S not indifferent any more at the last boundary point. We need to remove a step from the partition; hence smaller b results in smaller $N(b, u_R, u_S)$.²²

²¹In other words, given u_R , u_S and b , an ACT with $N^{CS}(b)$ steps exists in CS, but does not in this model. If we consider an ACT with $N^{CS}(b)$ steps in this model, at least one player deviates.

²²In this case, we take u_R and u_S to be that there is no ACT given b' as well as b .

Figure 8: Adjustment in a STOP in This Model

Of course, as in CS, smaller b can result in larger $N(b, u_R, u_S)$. Therefore, in this model, the smaller conflict of interest may lead to a smaller or larger maximum number of steps, as shown in Figure 9. The relationship between $N(b, u_R, u_S)$ and b is not monotonic.

Figure 9:
Non-Monotonicity in this model ($u_R=1/36, u_S=1/16$)

Proposition 10 shows the region in the $u_R u_S$ -plane where the non-monotonic relationship between information transmission $N(b, u_R, u_S)$ and conflict of interest b is observed.

Proposition 10 Fix any $u_R > 0$ and $u_S > 0$. Then, there exists an increasing continuous function $h(u_R)$ over $(0, 1/12]$ such that:

- (1) $N(b, u_R, u_S)$ is not monotonically decreasing in b for $u_S \in (h(u_R), \infty)$ and $u_R \in (0, 1/12)$.
- (2) Otherwise, $N(b, u_R, u_S)$ is monotonically decreasing in b .

Proof. See Appendix. ■

Figure 10 depicts Proposition 9. Recall that a STOP exists for $u_R \in (0, 1/12]$ and $u_S \in (0, \infty)$, and does not otherwise. Non-monotonicity is related to STOPs as we explained above.

Figure 10:
Existence of Non-Monotonicity between $N(b, u_R, u_S)$ and b

Figure 11 shows an example where there are both types of equilibria and $N(b, u_R, u_S)$ is monotonically decreasing in b .

Figure 11:
Monotonicity in This Model ($u_R=1/36$, $u_S=1/100$)

6.2 Welfare Implications of Non-Monotonicity

Non-monotonic relationships have interesting welfare implications.

Consider the best equilibrium for each player $j=R, S$, defined in Section 5. Let $U_j^*(b, u_R, u_S)$ denote the player j 's ex-ante expected payoffs in his best equilibrium.

As there is more conflict of interest over actions, measured by b , there can be an equilibrium with more steps, and hence the ex-ante expected payoffs can be larger for both players.

Lemma 11 $U_j^*(b, u_R, u_S)$ is not monotonically decreasing in b for $j=R$ and S .

Proof. See Appendix. ■

Section 5 shows that each player's expected payoffs are increasing with the number of steps among STOPS, and Section 6.1 shows the maximum number of steps is not always monotonically decreasing in the level of conflict of interest b when a STOP exists. Thus, we can obtain non-monotonic relationship between each player ex-ante expected payoff and conflict of interest b .

For example, consider $u_R = 1/24$ and $u_S = 81/64$. Compare different levels of conflict of interest $b = 1/50$, $b = 1/8$ and $b = 1$.

Given $b = 1/50$, ACTs exist, and the most informative ACT has five steps. The ex-ante expected payoffs are approximately -0.0065 for R and -0.0069 for S.

Given $b = 1/8$ and $b = 1$, ACT does not exist. Given $b = 1/8$, there only exists a babbling STOP. The ex-ante expected payoffs are $-u_R = -1/24 \approx -0.036$ for R and $-u_S = -81/64 \approx -1.265$ for S. On the other hand, given $b = 1$, there exists a STOP with two steps. The ex-ante expected payoffs are approximately -0.033 for R and -1.201 for S.

Both players' expected payoffs are maximized given $b = 1/50$. But if we compare $b = 1/8$ and $b = 1$, they are better off given $b = 1$.

Figure 12:
Effect of Non-Monotonicity on Welfare($u_R=1/24$, $u_S=81/64$)

If a manager can select an expert from a pool of experts including one with very small bias, the manager prefers the least biased expert. However, if the pool is limited and the manager cannot find a very unbiased expert, selecting the least biased expert from the limited expert pool may not be optimal for the manager.

7 Comparison with CS

7.1 Difference in the Maximum Number of Steps from CS

This section shows how the presence of an outside option affects communication. We compare information transmission between this model and CS. For this comparison, it suffices to consider STOPs because ACTs are equivalent to equilibria in CS.

The next lemma shows the comparison between the maximum number of steps in this model, $N(b, u_R, u_S)$, and that in CS, $N^{CS}(b)$.

Lemma 12 *Fix any $b > 0$, $u_R > 0$ and $u_S > 0$. Then, $N(b, u_R, u_S) \leq N^{CS}(b) + 1$.*

Proof. *See Appendix.* ■

This lemma provides an upper bound for the maximum number of steps that is above $N^{CS}(b)$. This upper bound is reached for some parameters where STOP exists. For example, consider $b = 1$ and $u_S = 81/64$. Take u_R such that $1/4 < \sqrt{12u_R} < 3/4$. In CS, the maximum number of steps is one (i.e., $N^{CS}(b) = 1$). In this model, the maximum number of steps is two (i.e., $N(b, u_R, u_S) = 2$) where the boundary point is at $\theta = 1/4$ and R selects inaction in the last step.

Why does this change in the maximum number of steps occur? Suppose S is very biased (i.e., b is large). Specifically, suppose b is just large enough that only a babbling equilibrium exists in CS. In this model, S can still convey information. To see this, suppose S uses one of two messages, "small" and "large," that R selects the outside option of inaction in response to "large." When the state is low, S prefers R's action in response to "low" to the outside option of inaction because the outside option is not very profitable for S. When the state is high, S prefers the outside option of inaction to R's action in response to "low" because R's action is too low. S honestly sends two messages separately dependent on the state.

We consider $u_R \in (0, 1/12]$ and $u_S \in (0, \infty)$ where a STOP exists and define three sets of (u_R, u_S) pairs, $LA(b)$, $EQ(b)$ and $SM(b)$, given b such that:

$$\begin{aligned} N^{STOP}(b, u_R, u_S) &= N^{CS}(b) + 1 && \text{for } (u_R, u_S) \in LA(b), \\ N^{STOP}(b, u_R, u_S) &= N^{CS}(b) && \text{for } (u_R, u_S) \in EQ(b), \\ N^{STOP}(b, u_R, u_S) &< N^{CS}(b) && \text{for } (u_R, u_S) \in SM(b). \end{aligned} \tag{23}$$

These sets show that the maximum number of steps among STOPs is larger, equivalent or smaller than the maximum number of steps in CS for the same b . $EQ(b)$ and $SM(b)$ exists for any b , and $LA(b)$ exists for some b .

Proposition 13 Take any integer $k \geq 1$. There exist cutoffs b_{k+1} , c_k and b_k , where $b_{k+1} < c_k \leq b_k$, such that²³:

- (1) For $b \in (b_{k+1}, c_k)$, $LA(b)$, $EQ(b)$ and $SM(b)$ are nonempty.
- (2) For $b \in (c_k, b_k)$, only $EQ(b)$ and $SM(b)$ are nonempty.

Proof. See Appendix. ■

Figures 13 and 14 illustrate Proposition 13. Take the conflict of interest b to be $N^{CS}(b) = k$ in CS. Then, in this model, $N^{STOP}(b, u_R, u_S) = k + 1$ for $(u_R, u_S) \in LA(b)$. As mentioned in Section 4, when both STOPs and ACTs exist, i.e., $u_R \in [C(b), 1/12]$ and $u_S \in (0, \infty)$, we observe $N^{ACT}(b) \geq N^{STOP}(b, u_R, u_S)$.

There are two important observations in Figure 13. For $(u_R, u_S) \in LA(b)$, only STOPs exist, and values of u_R and u_S are not extremely small or large. In other words, information transmission is improved when R's payoff from the outside option is relatively large, but every players' payoff from the outside option is not extremely small or large.

Figure 13: Comparison between $N^{STOP}(b, u_R, u_S)$ in this model and $N^{CS}(b)$ in CS given $b_{k+1} < b < c_k$

²³ b_k is derived from (13). Thus, in CS, $N^{CS}(b) = k$ if $b \in (b_{k+1}, b_k)$. We found the following relations: $0 < b_{k+1} < c_k < b_k < \infty$ for $k \geq 2$ and $0 < b_2 < c_1 = b_1 = \infty$. The formulas for b_k and c_k are given in the Appendix.

Figure 14: Comparison between $N^{STOP}(b, u_R, u_S)$ in this model and $N^{CS}(b)$ in CS given $c_k < b < b_k$

Figures 15 and 16 show $N(b, u_R, u_S)$ for a specific example. Consider the following two cases, (i) $b \in (1/12, 1/8)$ and (ii) $b \in (1/8, 1/4)$. In both cases, the maximum number of steps is two in CS, i.e., $N^{CS}(b) = 2$. On the other hand, in this model, the maximum number of steps can be three, i.e., $N(b, u_R, u_S) = 3$, for $(u_R, u_S) \in LA(b)$.

Figure 15: $N(b, u_R, u_S)$ in this model given $1/12 < b < 1/8$

Figure 16: $N(b, u_R, u_S)$ in this model given $1/8 < b < 1/4$

7.2 Welfare Comparison with CS

Section 7.1 shows that the presence of an outside option can increase information transmission compared to CS. This section explains how this increase in information transmission affects each player's welfare. We compare each player's ex-ante expected payoffs between this model and CS. This welfare comparison is different between R and S. This increase in information transmission is good for R but bad for S.

Recall that $N(b, u_R, u_S)$ and $N^{CS}(b)$ denote the maximum number of steps in this model and in CS, respectively. Recall that, taking the conflict of interest to be $b \in (b_{k+1}, c_k)$ for any integer k , there is a non-empty set $LA(b)$ such that $N(b, u_R, u_S) = N^{CS}(b) + 1$ for $(u_R, u_S) \in LA(b)$.

First, we take the standard approach of comparing ex ante expected payoffs. The results for R and S are given in Figures 17 and 18, respectively. In each figure, each player is better off in this model in the highlighted area and worse off in the dotted area than in CS.

R is better off when there is more information transmission (i.e., $(u_R, u_S) \in LA(b)$). R is also better off when the outside option is very profitable for him (i.e., u_R is small). On the other hand, S is worse off for $(u_R, u_S) \in LA(b)$. S is better off only if the outside option is very profitable for him (i.e., u_S is small).

Figure 17: Welfare Comparison for R

$b \in (b_{k+1}, c_k)$ for some $k \geq 1$. $N(b, u_R, u_S) = N^{CS}(b) + 1$ for $(u_R, u_S) \in LA(b)$.

Figure 18: Welfare Comparison for S

$b \in (b_{k+1}, c_k)$ for some $k \geq 1$. $N(b, u_R, u_S) = N^{CS}(b) + 1$ for $(u_R, u_S) \in LA(b)$.

Why is the welfare comparison different between R and S? One potential explanation is that the presence of an outside option has two effects on each player's welfare. The first one is an indirect communication effect. The presence of the outside option affects communication, which affects welfare. The second one is a direct effect, which is each player's payoff from the outside option. For $(u_R, u_S) \in LA(b)$, the communication effect dominates the direct effect for R but not for S.

To support this explanation, we consider a comparison which we call the "greenlighting comparison."²⁴ Define "greenlighting states" to be those in which an action is selected in a STOP with $N^{STOP}(b, u_R, u_S)$ steps in this model. By considering only states in which an action is chosen, this comparison eliminates the direct role of the outside options, leaving only the indirect effect on communication. Then, we compare each player's welfare across the greenlighting states between this model and CS. The greenlighting comparison is the same for the two players; both players are better off in this model than in CS for $(u_R, u_S) \in LA(b)$.

7.3 Other Non-Monotonic Relationships

Section 6 shows a non-monotonic relationship between the conflict of interest b and the maximum number of steps $N(b, u_R, u_S)$ given u_R and u_S . Comparison between this model and CS indicates other non-monotonic relationships. In particular, given b , $N(b, u_R, u_S)$ is not monotonically increasing in R's payoffs from the outside option $-u_R$ (or S's payoffs from the outside option $-u_S$). In other words, R may be able to collect more information by burning money – reducing his payoffs from the outside option.

²⁴We thank Andrew Newman for suggesting this approach.

Lemma 14 Fix $b > 0$. Then, $N(b, u_R, u_S)$ is not monotonically decreasing in u_R and u_S , respectively.

Proof. See Appendix. ■

For example, consider $b \in (1/12, 1/8)$. As shown in Figure 15, $N(b, u_R, u_S)$ is not monotonic in u_R fixing u_S . Specifically, the maximum number of steps is $N(b, u_R, u_S) = 3$ for $u_R < C(b)$ but $N(b, u_R, u_S) \leq 2$ for $u_R > C(b)$. $N(b, u_R, u_S)$ can also increase when u_R increases.

Consequently, each player's payoffs in his best equilibrium, $U_j^*(b, u_R, u_S)$, may not be monotonically increasing in R's payoffs from the outside option $-u_R$ (or S's payoffs from the outside option $-u_S$).

Lemma 15 Fix $b > 0$. Then, $U_j^*(b, u_R, u_S)$ is not monotonically decreasing in u_R and u_S , respectively for each player $j=R$ and S .

Proof. See Appendix. ■

A small decrease in u_R (or u_S) has almost no effect on each player's payoffs in the last step. However, the maximum number of steps $N(b, u_R, u_S)$ is not monotonic in u_R and u_S , respectively. Also, each player is better off in a STOP with more steps as shown in Section 5. Therefore, reducing u_R (or u_S) may result in a decrease or increase in $N(b, u_R, u_S)$, which may make both players worse off or better off.

8 Conclusion

There are three major insights to be gained from adding the outside option of inaction to the standard cheap talk model. Contrary to CS, conflict of interest can facilitate communication. As the level of conflict of interest between a sender and a receiver decreases, the sender's incentive to hide information may increase, which may result in less information transmission between them. Second, communication may be more informative than in CS. Third, more information transmission can make the sender worse off.

There are many interesting applications for this model. The first application includes managerial economics such as the design of contracts and organizations. The second includes political economy such as the design of legislative organization and the reform of redistributive policy. The third relates to the analysis of communication in markets.

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Appendix

Define a continuous function $g(u_R)$ and a step function $L(u_R)$ over $(0, 1/12]$:

$$L(u_R) := k \text{ such that } u_R \in \left(\frac{1}{3(k+1)^2}, \frac{1}{3k^2} \right), \quad \text{any integer } k \geq 2. \quad (24)$$

$$g(u_R) := \frac{L(u_R) \cdot \sqrt{12u_R} - 1}{2L(u_R) \cdot (L(u_R) - 1)}. \quad (25)$$

Define a cutoff $C(b)$:

$$C(b) := \left(\frac{2L(u_R) \cdot (L(u_R) - 1) \cdot b + 1}{L(u_R) \cdot \sqrt{12}} \right)^2, \quad (26)$$

where $C(b)$ is derived from $g(u_R)$ and $L(u_R)$ such that:

$$b > g(u_R) \Leftrightarrow u_R < C(b).$$

Define a function $h(u_R)$ over $(0, 1/12]$:

$$h(u_R) := \left(\frac{L(u_R) \cdot (L(u_R) - 2) \cdot \sqrt{12u_R} + 1}{2L(u_R) \cdot (L(u_R) - 1)} \right)^2 \quad (27)$$

Define b_N and c_N :

$$\begin{aligned} b_N &:= \frac{1}{2(N-1)N} \quad \text{for } N \geq 2, \\ c_N &:= \frac{1}{2(N-1)(N+2)} \quad \text{for } N \geq 2, \end{aligned} \quad (28)$$

and $c_1 = b_1 = \infty$. Thus, $0 < b_{N+1} < c_N < b_N < \infty$ for any integer $N \geq 2$.

It is useful to define the following sets, which are exclusive and exhaustive:

$$\begin{aligned} R_{STOP}(b) &:= \{(u_R, u_S) : 0 < u_R < C(b), 0 < u_S < \infty\}, \\ R_{BOTH}(b) &:= \{(u_R, u_S) : C(b) < u_R < 1/12, 0 < u_S < \infty\}, \\ R_{ACT}(b) &:= \{(u_R, u_S) : 1/12 < u_R < \infty, 0 < u_S < \infty\}. \end{aligned} \quad (29)$$

Hence, only STOPS exist for $(u_R, u_S) \in R_{STOP}(b)$, both STOPS and ACTs exist for $(u_R, u_S) \in R_{BOTH}(b)$, and only ACTs exist for $(u_R, u_S) \in R_{ACT}(b)$.

It is useful to define sets $Y(N; b)$ such that fixing b , a STOP with N steps exists if

$$(u_R, u_S) \in Y(N; b):$$

$$\begin{aligned} Y(1; b) &:= (0, 1/12] \times (0, \infty) \\ Y(N; b) &:= \{(u_R, u_S) : b \in B(N, u_R, u_S)\}. \end{aligned} \tag{30}$$

Thus, $(u_R, u_S) \in Y(k; b)$ if and only if:

$$b \cdot (2k - 3) < \sqrt{u_S} \leq \min \left\{ b + \sqrt{3u_R}, \frac{2(k-1)^2 b + 1 - \sqrt{12u_R}}{2(k-1)} \right\}.$$

$Y(N; b)$ is given by an area within an acute triangle in the u_R, u_S plane. Recall that fixing u_R and u_S , a STOP with N steps exists if $b \in B(N, u_R, u_S)$. Hence, $LA(b) = Y(N^{CS}(b) + 1; b)$, $EQ(b) = Y(N^{CS}(b); b)$ and $SM(b) = \bigcup_{N=1}^{N^{CS}(b)-1} Y(N; b)$.

Proof of Lemma 1

Let $a^j(\theta)$ denote an optimal action of player $j=R, S$ given θ , where $a^R(\theta) = \theta$ and $a^S(\theta|b) = \theta + b$. Let $U^j(a, \theta)$ denote the ex-post payoff of player $j=R, S$ given a and θ .

If inaction is not selected in equilibrium, it suffices to show the following claim.

Claim 16 *The set of actions induced in equilibrium is finite.*

Suppose S does not induce R to select inaction in any state. CS's Lemma 1 holds, that is, the equilibrium is given by a finite partition. Next, suppose S induces R to select inaction given some state. Because, for $j=R, S$, $U_1^j = 0$ for some a , $U_{11}^j < 0$ and $U_{12}^j > 0$, and because $a^S(\theta|b) \neq a^R(\theta)$ for all θ , the argument in CS's Lemma 1 applies; the set of actions induced in equilibrium is finite. Thus, Claim 16 holds.

If inaction is selected in equilibrium, it is useful to show the following claim.

Claim 17 *The set of states in which the same action is induced is connected.*

From Claim 16, if an action is induced in equilibrium, the support of the action is bounded. Because $U^R(a^R(\theta), \theta) > U^R(\emptyset, \theta)$ in any state, R does not select inaction under perfect information; the support of states inducing inaction should be nondegenerate. Suppose action a is induced given θ and θ' , where $\theta < \theta'$. This reveals that S prefers action a to inaction given θ and θ' ; S's utilities is strictly concave in θ ; S strictly prefers action a to inaction given any $\theta'' \in (\theta, \theta')$. Thus, we have

shown that a set of states in which S induces R to take the same action is connected. Thus, Claim 17 holds.

Hence, if S induces R to select inaction in equilibrium, its support can only include a disconnected set of states or/and finite degenerate states at boundary between elements inducing actions.

Proof of Lemma 2

Suppose S induces inaction at boundary point $\theta = \theta_{i-1}$ and induces an action given $\theta \in (\theta_{i-1}, \theta_i)$, where $\theta_{i-1} < \theta_i$. Because the set of states in which the same action is induced is connected, R takes an action $(\theta_{i-1} + \theta_i) / 2$ given a message $m \in (\theta_{i-1}, \theta_i)$. By continuity of S's utility function, S must be indifferent between inaction and action $(\theta_{i-1} + \theta_i) / 2$ at $\theta = \theta_{i-1}$; otherwise, S deviates and induces inaction if the true state is $\theta = \theta_{i-1} + \varepsilon$. Because $b > 0$, R's response is closer to S's optimal action given $\theta = \theta_{i-1}$ than $\theta = \theta_i$, i.e. $|(\theta_{i-1} + \theta_i) / 2 - a^S(\theta_{i-1})| < |(\theta_{i-1} + \theta_i) / 2 - a^S(\theta_i)|$. If S is indifferent between inaction and action $(\theta_{i-1} + \theta_i) / 2$ given $\theta = \theta_{i-1}$, there always exists $\varepsilon > 0$ such that S strictly prefers inaction to action $(\theta_{i-1} + \theta_i) / 2$ given $\theta \in (\theta_i - \varepsilon, \theta_i)$. This is a contradiction. Hence, when S induces R to take inaction given a set of states in equilibrium, this set should include $\theta = 1$. We have already shown that the set of states inducing inaction is nondegenerate. Thus, Lemma 2 holds. Q.E.D.

Proof of Proposition 3

From S's indifference conditions defined by (5), the width of the i -th element is increasing in i ; the wider element results in lower expected payoff for R; R's expected payoff is the lowest in the N -th element. R does not select inaction given any message if R does not given $m \in (\theta_{N-1}(N), \theta_N(N))$ because $(1 - \theta_{N-1}(N))^2 / 12 \leq 1/12$ for $u_R > 1/12$. Hence, the game is equivalent to CS. Q.E.D.

Proof of Proposition 4

Suppose that an ACT with N steps exists. R's optimality condition requires $\theta_N(N) - \theta_{N-1}(N) \leq \sqrt{12u_R}$, and S's indifference condition requires $\theta_N(N) - \theta_{N-1}(N) = 1/N + 2(N-1) \cdot b$. Hence, for existence, the following inequality should hold:

$$1/N + 2(N-1) \cdot b \leq \sqrt{12u_R}, \quad (31)$$

(31) derives $L(u_R)$ and $g(u_R)$ as defined in (24) and (25) such that $L(u_R) \leq N^{ACT}(b)$ for $b \leq g(u_R)$, and $L(u_R) > N^{ACT}(b)$ otherwise. Hence, for $b \leq g(u_R)$, there exists an ACT with N steps for every $N \in \{L(u_R), N^{ACT}(b)\}$. For $b > g(u_R)$, there is no ACT. Q.E.D.

Proof of Proposition 5

Consider $u_R \in (0, 1/12]$. A STOP with $N = 1$ step exists for any $b > 0$. Thus, $N^{STOP}(b, u_R, u_S)$ is well defined for any b, u_R and u_S .

Next we will show multiplicity. We need to derive sets $B(N, u_R, u_S)$, where $N \geq 2$. Suppose a STOP with $N \geq 2$ steps exists whose partition is denoted $\theta(N)$. S 's indifference conditions, (10) and (11), define $\theta_1(N)$:

$$\theta_1(N) = 2\sqrt{u_S} - 2b \cdot (2N - 3), \quad (32)$$

The first step should exist (i.e., $0 < \theta_1(N)$). Thus, the upper bound for b is defined:

$$b < \sqrt{u_S} / (2N - 3). \quad (33)$$

R 's optimality conditions, (12), require:

$$2\sqrt{u_S} - 2b \leq \sqrt{12u_R} \leq 1 - 2(k-1) \cdot \sqrt{u_S} + 2(k-1)^2 \cdot b, \quad (34)$$

which defines a lower bound for b :

$$\max \left\{ \sqrt{u_S} - \sqrt{3u_R}, \frac{\sqrt{12u_R} + 2(N-1)\sqrt{u_S} - 1}{2(N-1)^2} \right\} \leq b. \quad (35)$$

Hence, for $N \geq 2$, a set $B(N, u_R, u_S)$ is derived as defined in (14) in Section 4.

It suffices to show the following claim.

Claim 18 Consider $u_R \in (0, 1/12]$. Suppose that there exists a set $B(N, u_R, u_S)$ for some integer $N \geq 2$. Then, there exists a set $B(N-1, u_R, u_S)$ where $B(N, u_R, u_S) \subset B(N-1, u_R, u_S)$.

The claim holds for $N = 2$ because the definition of sets B implies $B(2, u_R, u_S) \subset B(1, u_R, u_S)$.

We will show that the claim holds for $N \geq 3$. Fix $b > 0$. Suppose that there exists a STOP with $N \geq 3$ steps whose partition is denoted $\theta(N)$. This partition

satisfies conditions (10), (11) and (12). Next, consider a partition $\theta(N-1)$ where $\theta_i(N-1) = \theta_{i+1}(N)$ for $i = 1, 2, \dots, N-2$. Conditions (10) and (11) are satisfied, and condition (12) is also satisfied. This partition $\theta(N-1)$ forms a STOP with $N-1$ steps for the same b . In addition, for $N \geq 2$, the upper bound of a set $B(N)$ is strictly decreasing in N . Hence, $B(N, u_R, u_S) \subset B(N-1, u_R, u_S)$.

Hence, the claim holds for $N \geq 2$. Q.E.D.

Proof of Lemma 7

First, we will show Lemma 7 (1). From the definition of each player's payoff, for any integer $N \geq 2$:

$$U_j^{ACT}(N) > U_j^{ACT}(N-1), \text{ where } j = R, S \Leftrightarrow b \in (0, b_N) \quad (36)$$

where $U_S^{ACT}(N) = U_R^{ACT}(N) - b^2$. Recall that an ACT with N steps exists only if $b \in (0, b_N)$. Thus, Lemma 7 (1) holds.

Next, we will show Lemma 7 (2). Consider $u_R \in (0, 1/12]$. It suffices to show that for any integer $N \geq 2$:

$$U_j^{STOP}(N) - U_j^{STOP}(N-1) > 0, \text{ where } j = R, S. \quad (37)$$

Consider R's payoff. For any integer $N \geq 2$:

$$U_R^{STOP}(N) - U_R^{STOP}(N-1) = \left(-\frac{(\theta_1^{STOP}(N)-0)^2}{12} + u_R \right) \cdot \theta_1^{STOP}(N) > 0, \quad (38)$$

where the first equality follows from that

$$\theta_i^{STOP}(N) - \theta_{i-1}^{STOP}(N) = \theta_{i-1}^{STOP}(N-1) - \theta_{i-2}^{STOP}(N-1)$$

for $i = 2, \dots, N-1$; the last inequality follows from:

$$0 < \theta_1^{STOP}(N) - 0 < \sqrt{12u_R}.$$

Consider S's payoff. For any integer $N \geq 3$:

$$\begin{aligned}
& U_S^{STOP}(N) - U_S^{STOP}(N-1) \\
&= \left(\frac{-(\theta_1^{STOP}(N)-0)^2}{12} + (u_S - b^2) \right) \cdot \theta_1^{STOP}(N) \\
&> \left(\frac{-(\theta_1^{STOP}(N)-0)^2}{12} + \left(1 - \frac{1}{(2N-3)^2} \right) \cdot u_S \right) \cdot \theta_1^{STOP}(N) > 0,
\end{aligned} \tag{39}$$

where the second inequality follows from $b \in (0, \sqrt{u_S}/(2N-3))$; the last inequality follows from $\theta_1(N) = 2(\sqrt{u_S} - b)$. For $N = 2$:

$$U_S^{STOP}(2) - U_S^{STOP}(1) = \left(\frac{-(\theta_1^{STOP}(2)-0)^2}{12} - b^2 + u_S \right) \cdot \theta_1^{STOP}(2) > 0, \tag{40}$$

where the last inequality follows from $\theta_1^{STOP}(2) - 0 = 2(\sqrt{u_S} - b) > 0$. Thus, Lemma 7 (2) holds. Q.E.D.

Proof of Propositions 8 and 9

Consider $(u_R, u_S) \in R_{BOTH}(b)$, i.e., both types of equilibria exist. From Lemma 7, it suffices to compare each player's ex-ante expected payoff in a STOP with $N^{STOP}(b, u_R, u_S)$ steps and that in an ACT with $N^{ACT}(b)$ steps.

Because sets Y are derived from sets B , as for sets B , sets Y have the following inclusion relationship:

$$Y(N+1; b) \subset Y(N; b) \text{ for any integer } N \geq 1. \tag{41}$$

Hence, $N^{STOP}(b, u_R, u_S)$ is given by:

$$N^{STOP}(b, u_R, u_S) = \arg \max_{N \geq 1} \{N : (u_R, u_S) \in Y(N; b)\}. \tag{42}$$

Next, we can define functions $\alpha(\cdot, \cdot, \cdot)$ and $\beta(\cdot, \cdot)$ as follows:

$$\begin{aligned}
& U_R^{STOP}(N) > U_R^{ACT}(k) \text{ if } u_R < \alpha(u_S, N, k), \\
& U_S^{STOP}(N) > U_S^{ACT}(k) \text{ if } u_S < \beta(N, k).
\end{aligned} \tag{43}$$

where $(u_R, u_S) \in Y(N; b)$, $b \in (0, b_k)$ and:

$$\alpha(u_S, N, k) := \sqrt{\frac{\frac{(k+1)(k-1)b^2}{3} + \frac{1}{12k^2} - \frac{2(N-1)(\sqrt{u_S} - (N-1)b)((\sqrt{u_S} - b(N-1))^2 + b^2N(N-2))}{3}}{1 + 2(N-1)^2b - 2(N-1)\sqrt{u_S}}},$$

$$\beta(N, k) := \left\{ u_S : \gamma(u_S, k, N) > 0, \sqrt{u_S} \in \left((2N-3)b, \frac{(N^2-2N+2)b+1/2}{N} \right) \right\},$$

$$\gamma(u_S, k, N) := \sqrt{u_S}^3 - \frac{3\sqrt{u_S}^2}{4(N-1)} - (2N^2 - 4N + 3)b^2\sqrt{u_S}$$

$$+ (N^2 - 2N + 2)(N-1)b^3 + \frac{(k^2+2)b^2}{4(N-1)} + \frac{1}{16(N-1)k^2}.$$

Consider R's ex-ante expected payoffs. It suffices to show:

$$u_R > \alpha(u_S, N, N^{ACT}(b)) \quad (44)$$

for $(u_R, u_S) \in Y(N, b) \cap R_{BOTH}(b)$.

This inequality holds as follows. First, $u_R > \alpha(u_S, N, k)$ holds for $N \leq k$ for $(u_R, u_S) \in R_{BOTH}(b)$. Second, $R_{BOTH}(b) \cap Y(N^{ACT}(b) + 1, b)$ is empty even if $Y(N^{ACT}(b) + 1, b)$ exists.

Consider S's ex-ante expected payoffs. Fix any $u_R \in [C(b), 1/12]$. Define $w(b) > 0$ such that:

$$w(b) := \beta(N, N^{ACT}(b)) \quad (45)$$

where $N = N^{STOP}(b, u_R, u_S)$ and:

$$w(b) \in (b, b(2N^{ACT}(b) - 3)) \text{ for } b \in (0, 1/12),$$

$$w(b) \in (b, b + 1/4) \text{ for } b \in [1/12, 1/4],$$

$$w(b) = 1/2 + b^2 \text{ for } b \in [1/4, \infty).$$

We can find $w(b) > 0$ for any b as follows. Consider $b \in (0, 1/12)$, i.e., $N^{ACT}(b) \geq 3$. We can take $u_S > \beta(N^{ACT}(b), N^{ACT}(b))$ for $(u_R, u_S) \in Y(N^{ACT}(b); b)$. We can also take sufficiently small u_S such that $u_S < \beta(2, N^{ACT}(b))$. Thus, there exists a cutoff $w(b) \in (b, b(2N^{ACT}(b) - 3))$, i.e., $(u_R, w(b)) \in Y(2; b) \setminus Y(N^{ACT}(b); b)$. That is, for $u_S < w(b)$, $N^{STOP}(b, u_R, u_S) < N^{ACT}(b)$ holds, and S's ex-ante best equilibrium is a STOP with $N^{STOP}(b, u_R, u_S)$ steps. Similarly, consider $b \in [1/12, 1/4]$, i.e., $N^{ACT}(b) = 2$. We can find $w(b) \in (b, b + 1/4)$, i.e., $(u_R, w(b))$ belongs to interior of a set $Y(2; b)$. For $b \in [1/4, \infty)$, i.e., $N^{ACT}(b) = 1$, we can find $w(b) =$

$\beta(1,1) = 1/2 + b^2$. In summary, for any b , we can take sufficiently small u_S such that $u_S < w(b)$ and $N^{STOP}(b, u_R, u_S) = 1$, i.e., S 's ex-ante best equilibrium is a babbling STOP. Q.E.D.

Proof of Proposition 10

Consider $u_R \in (0, 1/12]$. From definitions of sets B and functions L and g , the following inequality holds for $u_R \in (0, 1/12]$ and $u_S \in (0, \infty)$:

$$g(u_R) < \inf_{b \in B(L(u_R)+1)} b \quad (46)$$

For $b \in (0, g(u_R))$, both STOPs and ACTs exist, and (46) implies $N^{ACT}(b) \geq N^{STOP}(b, u_R, u_S)$. For $b \in (g(u_R), \infty)$, only STOPs exist. Hence, the maximum number of steps $N(b, u_R, u_S)$ is well defined such that:

$$\begin{aligned} N(b, u_R, u_S) &= N^{ACT}(b) \text{ for } b \in (0, g(u_R)), \\ N(b, u_R, u_S) &= N^{STOP}(b, u_R, u_S) \text{ for } b \in (g(u_R), \infty) \end{aligned} \quad (47)$$

Recall that $N^{ACT}(b)$ is decreasing in b , and that $N^{ACT}(b) = L(u_R)$ if $b = g(u_R) - \epsilon$ at the limit $\epsilon \searrow 0$. From Claim 14, $N^{STOP}(b, u_R, u_S)$ is increasing in b if $b < \inf_{b \in B(N^{STOP}(b, u_R, u_S))} b$ and decreasing in b otherwise.

Hence, if $g(u_R) < \inf_{b \in B(L(u_R))} b$ holds, we can take $\epsilon > 0$ such that:

$$\begin{aligned} N(b, u_R, u_S) &= N^{ACT}(b) = L(u_R) \text{ for } b \in (g(u_R) - \epsilon, g(u_R)), \\ N(b, u_R, u_S) &= N^{STOP}(b, u_R, u_S) < L(u_R) \text{ for } b \in (g(u_R) - \epsilon, g(u_R) + \epsilon), \\ N(b, u_R, u_S) &= N^{STOP}(b, u_R, u_S) \geq L(u_R) \text{ for } b \in \left(\inf_{b \in B(L(u_R))} b, \sup_{b \in B(L(u_R))} b \right), \end{aligned} \quad (48)$$

which implies that $N(b, u_R, u_S)$ is decreasing in b over $b \in (0, g(u_R))$, increasing in b over $b \in \left(g(u_R), \inf_{b \in B(N^{STOP}(b, u_R, u_S))} b \right)$, and is decreasing in b over $b \in \left(\inf_{b \in B(N^{STOP}(b, u_R, u_S))} b, \infty \right)$. Hence, $N(b, u_R, u_S)$ is non-monotonic in b if and only if $g(u_R) < \inf_{b \in B(L(u_R))} b$ holds, where:

$$g(u_R) < \inf_{b \in B(L(u_R))} b \Leftrightarrow u_S > h(u_R) \quad (49)$$

Hence, $N(b, u_R, u_S)$ is non-monotonic in b if and only if $u_S > h(u_R)$. Q.E.D.

Proof of Lemma 11

Let $U_j^{STOP}(N, b, u_R, u_S)$ denote the ex-ante expected payoffs in a STOP with N steps for player $j = R, S$ given by equations (16), (17) and (18). $U_j^{STOP}(N, b, u_R, u_S)$ is decreasing in b and increasing in N .

Proposition 10 implies that for $u_S \in (h(u_R), \infty)$ and $u_R \in (0, 1/12)$, we can find b such that $N^{STOP}(b, u_R, u_S) < N^{STOP}(b + \epsilon, u_R, u_S)$ for any $\epsilon > 0$ (i.e., b belongs to the boundary of a set $B(N, u_R, u_S)$ for some N). Take such a parameter b and let $N^{STOP}(b, u_R, u_S) = N$. We can find $\epsilon > 0$ such that:

$$\begin{aligned} & U_j^*(b + \epsilon, u_R, u_S) - U_j^*(b, u_R, u_S) \\ &= \left(U_j^{STOP}(N + 1, b, u_R, u_S) - U_j^{STOP}(N, b, u_R, u_S) \right) \\ & \quad + \left(U_j^{STOP}(N, b + \epsilon, u_R, u_S) - U_j^{STOP}(N, b, u_R, u_S) \right) \\ &> 0 \end{aligned} \tag{50}$$

because $U_j^{STOP}(N + 1, b, u_R, u_S) - U_j^{STOP}(N, b, u_R, u_S) > 0$ and $U_j^{STOP}(N, b + \epsilon, u_R, u_S) - U_j^{STOP}(N, b, u_R, u_S) \nearrow 0$ as $\epsilon \searrow 0$. We can also find $\epsilon' > 0$ such that $N^{STOP}(b - \epsilon', u_R, u_S) = N^{STOP}(b, u_R, u_S)$ where $U_j^*(b, u_R, u_S) > U_j^*(b - \epsilon', u_R, u_S)$. Thus, $U_j^*(b, u_R, u_S)$ is decreasing in b over $(b - \epsilon', b)$ and increasing in b over $(b, b + \epsilon)$. Q.E.D.

Proof of Lemma 12

Fix $b > 0$. Suppose that a STOP exists and $N^{STOP}(b, u_R, u_S) \geq N^{CS}(b) + 2$. The definition of $N^{CS}(\cdot)$ implies that for any $\theta_1 > 0$:

$$\sum_{i=1}^{N^{CS}(b)+1} (\theta_1 + 4(i-1) \cdot b) > 1. \tag{51}$$

This implies that for any $\theta_1 (N^{STOP}(b, u_R, u_S)) > 0$:

$$\sum_{i=1}^{N^{STOP}(b, u_R, u_S)-1} \theta_i (N^{STOP}(b, u_R, u_S)) - \theta_{i-1} (N^{STOP}(b, u_R, u_S)) > 1, \tag{52}$$

where $\theta_i (N^{STOP} (b, u_R, u_S)) - \theta_{i-1} (N^{STOP} (b, u_R, u_S)) = \theta_1 (N^{STOP} (b, u_R, u_S)) + 4 (i - 1) \cdot b$. This contradicts the definition of an equilibrium partition. Q.E.D.

Proof of Proposition 13

The definition of sets $Y (N; b)$ in (30) implies $LA (b) = Y (N^{CS} (b) + 1; b)$, $EQ (b) = Y (N^{CS} (b); b)$ and $SM (b) = \bigcup_{N=1}^{N^{CS}(b)-1} Y (N; b)$.

It suffices to show that $Y (N; b)$ is non-empty for $N \in \{1, 2, \dots, N^{CS}(b) + 1\}$. From the definition, we can show that $Y (N^{CS}(b) + 1; b)$ is non-empty only for $b \in (b_{N+1}, c_N)$ while $Y (N; b)$ is always non-empty for any $N \leq N^{CS}(b)$ for any b .

We can also show $Y (N^{CS}(b) + 1; b) \subset R_{STOP} (b)$, but $Y (N; b) \cap R_{STOP} (b) \neq \emptyset$ and $Y (N; b) \cap R_{BOTH} (b) \neq \emptyset$ for any $N \leq N^{CS}(b)$.

Therefore, $LA (b)$ is not empty only in the region $R_{STOP} (b)$ for $b \in (b_{N+1}, c_N)$ for some N ; $LA (b)$ does not exist for $b \in (c_N, b_N)$ for some N . Q.E.D.

Proof of Lemmas 14 and 15

Proposition 13 implies that for any b , we can find (u_S, u_R) such that $N^{STOP} (b, u_R, u_S) < N^{STOP} (b, u_R + \epsilon, u_S)$ for any $\epsilon > 0$ or $N^{STOP} (b, u_R, u_S) < N^{STOP} (b, u_R, u_S + \epsilon)$ for any $\epsilon > 0$ (i.e., (u_S, u_R) belongs to the boundary of a set $Y (N; b)$ for some N). Thus, Lemmas 14 and 15 are shown applying the approaches in Proof of Lemma 11. Q.E.D.