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Extracting Implied Dividends from Options Prices: some Applications to the Italian Derivatives Market

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Abstract. This contribution deals with options on assets which pay discrete dividends. We analyze some methodologies to extract information on dividends from observable option prices. Implied dividends can be computed using a modified version of the well known put-call parity relationship. This technique is straightforward, nevertheless, its use is limited to European options and, when dealing with equities, most traded options are of American-type. As an alternative, numerical inversion of pricing methods can be used. We apply different procedures to obtain implied dividends of stocks of the Italian Derivatives Market.

Keywords: Implied dividends, put-call parity, option pricing, binomial methods.

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1 Introduction

Stock options are normally unprotected from cash dividends paid on the underlying. Dividend payments during the option's life reduce the stock price by an amount proportional to the size of the dividend and hence reduce (increase) the value of call (put) options. In the event of extraordinary cash dividends, the Options Clearing Corporation protects the value of options by adjusting the exercise prices. When considering aggregated dividends, which is the case when dealing with indexes, one can assume that the uncertainty is balanced out, but stock options can be affected by a single cash dividend; thus, a change in the latter has a significant impact on the options prices.

In this contribution we propose and analyze some methodologies to extract information on dividend uncertainty from observable option prices. A fundamental aspect when valuing index and stock options correctly is the knowledge of the amount and the timing of the cash dividends that will be paid before the option expiration. Usually, derivative pricing theory assumes that dividends are known both in size and timing. However, this assumption might be too strong.

Put-call parity describes the relationship that must occur between the price of European call and put options to prevent arbitrage. Such a relation is independent of a pricing model and, therefore, it can be used to test the market efficiency. First note that for each pair of European call and put options with the same strike and maturity, implied dividends can be computed using a modified version of the well known parity relationship. Harvey and Whaley [3] and Brooks [2] use the parity to predict dividends on S&P Index and single stocks. This technique is straightforward and does not depend on the assumptions about the underlying price process dynamics. Nevertheless, its use is limited to European options. As an alternative, numerical inversion of pricing methods, such as an interpolated binomial approach analyzed in [7], can be used to derive implied dividends from market data. We apply such a procedure to obtain implied dividends of stocks in FTSEMIB index.

The remainder of this paper is structured as follows. The next section considers extracting cash dividends using put-call parity. Section 3 describes the single stock case. In Section 4 experimental analysis is reported and some conclusions are drawn.

2 Cash dividends predictions using put-call parity

In this section we analyze the ability of the well-known put-call parity relationship to predict future cash dividends. Using no arbitrage arguments it is easy to prove the following relationship which relates the price of a European call option and an otherwise identical put to the value of a synthetic bond, the value of the underlying and the present value of the dividend that the asset pays until the expiration of the options. For the single dividend case, the put-call parity relation is

$$c_0 - p_0 = S_0 - D e^{-rt_D} - X e^{-rT}, \quad (1)$$

where: D is the cash dividend¹ paid at time t_D , S_0 is the current asset price, r is annualized continuously computed risk-free rate of interest, p_0 is the current premium for a European put option, c_0 is the current premium for a European call option, T is the time (in years) to expiration, X is the option strike price.

Theoretically, one can obtain the implied dividend D in equation (1) from any pair of option premia with the same strike and maturity. All we need is a pair of simultaneous option prices, the current stock value, an estimate of the interest rate and the knowledge of the date of dividend payment. Implementing the put-call relationship faces several theoretical and practical problems that can be mitigated using particular procedures.

Firstly, it is worth emphasizing that although most index options are of European type, options on single stock are normally of American type. Many empirical studies test the put-call parity both for European and American options. As well known, parity (1) does not hold for American options, due to the possibility of early exercise, which cannot be completely ruled out when the strategies are established. To avoid the early exercise problem only European options must be considered. If the options are of American style, the following double inequality holds

$$S_0 - D e^{-rt_D} - X \leq C_0 - P_0 \leq S_0 - D e^{-rt_D} - X e^{-rT}, \quad (2)$$

where C_0 and P_0 are the current prices of an American call and put option, respectively. Note that inequalities (2) can be used to obtain an upper bound for the expected dividend

$$D e^{-rt_D} \leq S_0 + P_0 - C_0 - X e^{-rT}. \quad (3)$$

Regarding the optimal exercise of American call options, it is easy to prove that early exercise can be convenient just before a dividend payment. If the amount of the dividend is less than the time value of the strike price, $D < X [1 - e^{-r(T-t_D)}]$, then it is never convenient to exercise the call option before the expiration. As a result, also in presence of dividends we have $c_0 = C_0$. For American put options, early exercise may be optimal even in the absence of dividends and normally the inequality $P_0 \geq p_0$ is strictly verified.

Secondly, the practical use of the put-call relationship requires an estimation of the risk-free rate. Options literature employs either LIBOR or treasure T-note rate. The T-note are the most traded safe investments, but only the Governments can borrow at this rate. On the other hand, LIBOR rate can be subject to credit risk. Therefore, it is not totally clear which interest rate one can use in order to implement the model.

A third problem concerns the bid-ask quote convention for trading stocks and options. To mitigate the noise introduced by the bid-ask spread one can use the quote midpoints. Another problem regards the necessity to transform the index points into dividends payed on the single stocks. It is clear that overcoming all these issues is a difficult task.

3 Valuing equity options with cash dividends

As in the previous section, we assume that dividends are a pure cash amount D to be paid at a specified date t_D . Empirically, one observes that at the ex-dividend date the stock

¹In the case of multiple dividends, $D e^{-rt_D}$ is replaced by the sum of the present values of the future dividends.

price drops: in order to exclude arbitrage opportunities, the jump in the stock price should be equal to the size of the net dividend. Dividend payments during the life of the option imply lower call and higher put premia.

Dividends affect option prices through their effect on the underlying stock price. Since in the case of cash dividends we cannot use the proportionality argument, the price dynamics depends on the timing of the dividend payment. In a continuous time setting, the underlying price dynamics is assumed to satisfy the following stochastic differential equation

$$\begin{aligned} dS_t &= rS_t dt + \sigma S_t dW_t \quad t \neq t_D, \\ S_{t_D} &= S_{t_D}^- - D_{t_D}, \end{aligned} \quad (4)$$

where $S_{t_D}^-$ and S_{t_D} denote the stock price an instant before and after the jump at time t_D , respectively. Due to this discontinuity, the solution to equation (4) is no longer lognormal but in the form

$$S_t = S_0 e^{(r-\sigma^2/2)t + \sigma W_t} - D_{t_D} e^{(r-\sigma^2/2)(t-t_D) + \sigma W_{t-t_D}} I_{\{t \geq t_D\}}, \quad (5)$$

where I_A denotes the indicator function of A .

Haug *et al.* [4] (henceforth HHL) derived an exact expression for the fair price of a European call option on a cash dividend paying stock. The basic idea is that after the dividend payment, option pricing reduces to simple Black-Scholes (BS) formula for a non-dividend paying stock. Before t_D one considers the discounted expected value of the BS formula adjusted for the dividend payment. In the geometric Brownian motion setup, the HHL formula for a European call option is

$$c_{HHL}(S_0, T; D, t_D) = e^{-rt_D} \int_d^\infty c_{BS}(S_x - D, T - t_D) \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx, \quad (6)$$

where

$$\begin{aligned} d &= \frac{\log(D/S_0) - (r-\sigma^2/2)t_D}{\sigma\sqrt{t_D}}, \\ S_x &= S_0 e^{(r-\sigma^2/2)t_D + \sigma\sqrt{t_D}x}, \end{aligned}$$

and $c_{BS}(S_x - D, T - t_D)$ is given by the BS formula with time to maturity $T - t_D$. The price of a European put option with a discrete dividend can be obtained by exploiting put-call parity results.

For an American call option, since early exercise may be optimal only an instant prior to the ex-dividend date, one can merely replace relation (6) with

$$C_{HHL}(S_0, T; D, t_D) = e^{-rt_D} \int_d^\infty \max\{S_x - X, c_{BS}(S_x - D, T - t_D)\} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx. \quad (7)$$

For American put options, early exercise may be optimal even in the absence of dividends. Since no analytical solutions for both the option price and the exercise strategy are available, one is generally forced to numerical solutions, such as lattice approaches. The evaluation of options using binomial methods entails some numerical difficulties when the

underlying asset pays one or more discrete dividends, due to the fact that the tree is no longer recombining.

A method which performs very efficiently and can be applied to both European and American call and put options is a binomial method² which maintains the recombining feature and is based on an interpolation idea proposed by Vellekoop and Nieuwenhuis [8] (see also Nardon and Pianca [7] for an analysis of this and alternative approaches). Such a method can be easily extended to the valuation of option with multiple dividends, which is relevant when evaluating options on indexes.

The procedure can be described as follows: a binomial tree is constructed without considering dividends (with $S_{ij} = S_0 u^j d^{i-j}$, $u = e^{\sigma\sqrt{T/n}}$, and $d = 1/u$), then it is evaluated by backward induction from maturity until a dividend payment; at the node corresponding to an ex-dividend date (at step n_D), we approximate the continuation value V_{n_D} using the following linear interpolation³

$$V(S_{n_D,j}) = \frac{V(S_{n_D,k+1}) - V(S_{n_D,k})}{S_{n_D,k+1} - S_{n_D,k}} (S_{n_D,j} - S_{n_D,k}) + V(S_{n_D,k}), \quad (8)$$

for $j = 0, 1, \dots, n_D$ and $S_{n_D,k} \leq S_{n_D,j} \leq S_{n_D,k+1}$; then continue backward along the tree. Negative prices may arise in some cases, in particular when dividends are high. As a solution, one can impose an absorbing barrier at zero when the dividend is higher than the underlying price (dividends are not fully paid due to limited liability).

4 Implied dividends

Besides information about the distribution of the underlying price and its volatility, inference about dividend payouts can be derived from market data. Option pricing theory usually assumes that stocks pay known dividends, both in size and timing. Moreover, new dividends are often supposed to be equal to the former ones. As already pointed out, these assumptions are strong and not realistic. In this work we assume that the time in which dividends are paid is announced, but their amount is unknown. The aim is to derive implied dividends from market information about option prices.

Let us observe that dividend policies are not uniform for all traded assets. With reference to the Italian market (in particular we consider stocks in the FTSEMIB index), there are some firms that pay no dividend at all (this choice has been justified by the recent financial crisis) and firms that during the year pay dividends once, twice or even quarterly. Dividends can be paid in cash (normally in euro, but sometimes in dollars, hence one has to evaluate currency risk) or alternatively by issuing new shares of stock (in a number which is proportional to the shares already held) or warrants, or could be a mixture of stocks and cash. Taking into account in the evaluation model all such different dividend policies is a tough task.

²The interpolation procedure here described can be applied also to other numerical schemes, such as finite difference schemes for the pricing of European and American options.

³Other interpolation schemes can be considered.

If dividends are announced, formulas (6) and (7) can be used to derive implied volatilities from market data⁴. It is worth noting that the computation and numerical inversion of (6) and (7) entail some drawbacks concerning the approximation of the integral in order to obtain accurate results. In particular, difficulties arise when considering dividends paid very near in the future or very close to the option's maturity. Truncation of the integral interval has also to be chosen carefully. As an alternative, we also used the binomial method based on interpolation (8).

Due to the computational efforts required by the method, and the fact the dividend policies are differentiate, one may wonder if it is possible to obtain implied volatilities which are not model-based, but derived using only market price of traded options (see, for instance, [5]). Along this line, a procedure which computes a volatility index is that used by CBOE for the calculation of VIX, which provides a measure of the expected stock market volatility over the next 30 calendar days.

In the case of unknown dividends, formulas (6) and (7) and the numerical procedure described in section 3 can be used to derive implied dividends from market data. We apply such a procedure to obtain implied dividends of the stock prices in FTSEMIB index; some empirical results are reported in the next section.

By equating the observed market prices and the corresponding theoretical option values, we have to solve an equation in two unknowns: the implied volatility and the implied dividend. We then fix the volatility by using a model-free implied volatility $\hat{\sigma}$ obtained with a procedure similar to VIX, based on a set of at-the-money and out-of-the-money call and put options in the two nearest-term expiration months⁵.

5 Empirical experiments

In this section our aim is to draw information on the future dividends by analyzing the prices of both single stocks and index options traded on the IDEM. We assume that a set of option prices is observed, which contain all relevant information concerning the underlying asset.

We introduce a short empirical study based on options written on FTSE MIB. The FSTE MIB index is a recent re-branded of the S&P MIB index. The FSTE index options are of European style; the quotations are in index points and the value of one index point (multiplier) is 2.50 euro. There are at least 15 price levels with interval of 500 index points for the series with remaining life shorter than one year; at least 21 price levels with interval of 1 000 index points for the series with remaining life longer than one year. At the same time in each section, the negotiable expirations are the four quarterly expirations (March, June, September, December), the two nearest monthly expirations and the four six-months maturities (June and December) of the two years subsequent the current year, for a total of ten expirations. New issued options are quoted on the first trading day following expiration.

⁴In particular, formula (7) can be numerically inverted in order to compute the implied volatilities from the prices of American equity options; some results of empirical experiments on options of the Italian Derivatives Market (IDEM) are reported in [6].

⁵Calibration can be an alternative solution to the problem.

Table 1: Implied dividends of European call and put options on FTSEMIB (with $S_t = 22506$, $t = 11$ March 2010, $t_D = 24$ May 2010, $T = 18$ June 2010, $r = 0.0064$, $\hat{\sigma} = 0.23899432$ computed on options expiring in April and May)

X	call (bid-ask av.)	put (bid-ask av.)	D (put-call par.)	implied dividend
19500	2572.5	294.0	762.31	785.54
20000	2510.0	376.0	407.22	496.64
20500	2115.0	482.0	409.09	445.56
21000	1750.0	612.5	405.45	408.09
21500	1412.5	775.0	406.32	395.63
22000	1110.0	970.0	404.68	402.53
22500	845.0	1205.0	405.55	435.33
23000	625.0	1485.0	406.42	489.36
23500	449.0	1807.5	405.78	559.08
24000	314.0	2165.0	399.14	635.79
24500	217.0	2577.5	409.52	721.30

The expiration day is the third Friday of the month in which the option expires. The exercise at maturity for in-the-money options is automatic and settled in cash. The trading hours are: from 9:00 a.m. to 5:40 p.m. (during expiration day: from 9:00 to 9:05 a.m.).

The empirical analysis relates to quotations at the trading day 11 March 2010; at that time that there are some news on future dividends. We considered put and call options that expire in June and dividends paid the 24 May (time is computed in years, considering calendar days). Numerical results are reported in table 1, which compares implied dividend obtained using put-call parity and numerical inversion of HHL formula⁶. We have calculated the bid-ask average for the call and put prices. Dividend in the last column are computed as an average between dividends obtained from put and call options. The two approaches yield implied dividends (measured in index points) which are similar for strikes near at-the-money. Put-call parity violations are due to various reasons, among which we mention the fact that we do not take into account taxation (see e.g. [1] for a study on the Australian market). The more variability of the dividend in the second approach can be explained as follows: when a model with constant volatility is considered, we have a sort of smile effect for the dividend.

As a second experiment, we focused on American options. First we have considered American call and put options written on ENI stock, with maturity June 2010. Figure 1 shows the dividends obtained using the interpolated binomial method, based on a set of put option prices. We have also computed the implied dividend from American call and put options written on Italcementi stock, with maturity June 2010. Figure 2 shows the dividends obtained using the interpolated binomial method, based on a set of put option prices. In both examples $\hat{\sigma}$ has been computed using a set of at-the-money and out-of-

⁶Alternatively, an interpolated binomial method with 1000 or 2000 steps provides the same results. Moreover, such method is very fast.)

the-money options expiring in April and May. It is interesting to observe the behavior of implied dividends, which show a smile effect in the case of ENI put options and a more skewed shape in the latter case.

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Figure 1: Dividend predictions on American put options on ENI ($S_t = 17.78$, $t = 11$ March, $r = 0.0064$, $t_D = 0.202740$, 24 May, $T = 0.271233$, 18 June, $\hat{\sigma} = 0.20022985$)

Figure 2: Dividend predictions on American put options on Italcementi ($S_t = 8.9$, $t = 11$ March, $r = 0.0064$, $t_D = 0.202740$, 24 May, $T = 0.271233$, 18 June, $\hat{\sigma} = 0.28514498$)