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A credit contagion model for the dynamics of the rating transitions in a SME bank loan portfolio

Antonella Basso and Riccardo Gusso

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Abstract

In this work we analyze the effects of credit contagion on the credit quality of a portfolio of bank loans issued to SMEs. To this aim we start from the discrete time model proposed in Barro and Basso (2005), that considers the counterparty risk generated by the business relations in a network of firms, and we modify it by introducing different rating classes in order to manage the case of firms with different credit qualities. The transitions from a rating class to another occurs when a proxy for the asset value of the firm crosses some rating specific thresholds. We assume that the initial rating transition matrix of the system is known, and compute the thresholds using the probability distribution of the steady state of the model. A wide Monte Carlo simulation analysis is carried out in order to study the dynamic behaviour of the model, and in particular to analyze how the default contagion present in the model affects the output rating transition matrix of the portfolio.

Keywords: credit risk, counterparty risk, contagion models, bank loan portfolios, rating transitions, Monte Carlo simulation.

JEL Classification Numbers: G33, G21, C15

MathSci Classification Numbers: 65C05, 90B99

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1. Introduction

In this paper we study the effects of credit contagion on the credit quality of a portfolio of bank loans; in particular we investigate how credit contagion can affect the credit quality downgrade/upgrade of the firms in the portfolio. As it is done in practice, we identify the credit quality of a firm with a “rating” associated to it, as the ones assigned by rating agencies such as Standard & Poor’s or Moody’s or obtained by internal bank rating systems, and we investigate the downgrades and upgrades of the ratings of the firms in the portfolio in an dynamic discrete-time setting by means of the dynamic rating transition matrix.

A number of different approaches have been recently proposed in the literature for modelling the credit risk of a portfolio of bank loans; see for example Giesecke and Weber (2004): Frey and Backhaus (2004): Egloff, Leippold and Vanini, (2007): Neu and Kühn (2004). Among these different approaches, the counterparty risk model proposed in Barro and Basso (2005) models the asset value of a firm following a structural approach, and can be generalized in such a way as to take into account the presence of different rating classes.

In such a model, a proxy $V_i$ for the asset value of firm $i$ is described as the sum of three terms: a macroeconomic component which considers the influence of the business cycle through a factor model, a microeconomic component which models the business connections with other firms, and a residual idiosyncratic random term. The microeconomic component takes into consideration the direct business connections between the firms in the bank portfolio and their clients, and explains how the default of a client may cause financial distress to its suppliers and a possible downgrade of their credit quality. In this way a contagion mechanism is introduced in the model.

We consider a portfolio of bank loans issued to small and medium-sized enterprises (SMEs), and assume that they have been assigned a rating class that reflects their credit quality. We estimate from historical data, using a maximum likelihood method under a time homogeneity assumption, an
initial rating transition matrix for the system, whose elements are the probabilities of the transition from a rating class to another. The transition of a firm from a rating class to another occurs when the value $V_i$ crosses some rating specific thresholds which are computed using the probability distribution of the steady state of the model. In such a way the model enables to describe the evolution in time of the ratings of all the firms in the portfolio.

In order to analyze how the default contagion affects the system and influences the credit quality of the firms in the portfolio, we apply a Monte Carlo simulation technique and carry out a wide simulation analysis. In particular we simulate the behaviour of the model for different values of the parameters on a 10-year time horizon, and we analyze the results obtained for the defaults and the rating transitions of the firms in the portfolio.

The paper is structured as follows: in Section 2 we present a brief review of the literature on counterparty risk and contagion models; in Section 3 we present the model proposed, which generalizes that presented in Basso and Barro (2005) and allows to model the rating transitions of the portfolio positions year after year. In section 4 we describe the simulation procedure applied in the empirical analysis and discuss the results obtained. Finally, Section 5 presents some concluding remarks.

2. Counterparty risk and credit contagion models

In the most popular models for credit risks, both in reduced form and structural models, the dependence among the defaults and the credit quality downgrades of different firms is modelled using some state variables which represent the major macroeconomic factors. In the reduced form models the default intensities depend on these factors, while in the structural models it is the asset value of firms in the portfolio that depends on them. In both approaches, the common dependence on these macroeconomic variables, which reflect the state of the economy and the business cycle, introduces some dependence in the rating transitions and in the default probabilities.
Nevertheless, some recent empirical results have pointed out that the dependence on common macroeconomic factors fails to explain properly the clustering of defaults observed when the economy is in a recession period; see for example Jarrow and Yu (2001) and Das, Duffie and Kapadia (2005). This suggests that a firm-specific risk term could be introduced, which accounts for the changes in the firms’ health due to some microeconomic effect, as for example that generated by the business relations with firms’ counterparties.

To this purpose, Jarrow and Yu (2001) introduced the notion of counterparty risk, defined as the risk that the default of a firm’s counterparty affects its default probability. In a wider sense the counterparty risk can be defined as the risk that the default of a client causes a change in the credit quality of a firm. If the firms in a portfolio are strongly interdependent in terms of their business relations, as it is often the case in portfolios of bank loans issued to SMEs operating in the same geographical area, then the counterparty risk may play an important role; in this case the default of one firm induces a contagion effect on other firms through the network of the business relations, which can lead to the deterioration of their credit quality and even to their default.

Afterwards, several recent papers have introduced a counterparty risk term to model a microeconomic dependence in terms of direct inter-firm relationships, often jointly with the dependence on the business cycle. Along this line, Giesecke and Weber (2004) presents a model in which firms interact with their business partners in a lattice-type economy. Here the contagion effect is modelled as liquidity shocks generated when some counterparties fail to honour their obligations; firms in the economy jump from a “good” state to a “bad” one, and vice versa, with an intensity that is proportional to the number of their counterparties in the opposite state. The empirical investigation of this model shows that the contagion process leads to additional fluctuations of the portfolio losses around their averages.

In Egloff, Leippold and Vanini (2007), microstructural data obtained from a bank’s credit risk department are used to build a topological risk map of the
bank’s credit portfolio, which is represented by a weighted graph connecting firms in the portfolio, where the weights are related to the business relations between the firms. Then Monte Carlo simulation is used to analyze the effects of different interdependence microstructures on the correlation structure and on the risk figures of the credit portfolio; their findings show that the tail behaviour of the portfolio credit losses is significantly modified by the presence of the contagion effect.

Neu and Kühn (2004), in analogy with a lattice gas model borrowed from physics, models the correlations between sequential defaults by introducing functionally defined couplings between mutually dependent counterparties. The paper focuses on the estimation of the impact of the counterparty risk on the capital allocations in loan portfolios; the outcomes obtained by a simulation analysis of the model again suggest that corporate dependency introduces an additional source of risk and can significantly amplify the portfolio losses.

3. Modelling credit contagion and rating transitions in a portfolio of bank loans

The main goal of this contribution is to propose a model that allows to study the effects of the counterparty risk not only on the clustering of defaults in a SME bank loan portfolio, but also on the co-movements of the credit quality of firms. To this aim, let us relate the credit quality of a firm $i$ (for $i=1,...,N$), to the value of a proxy for the firm’s asset value at time $t$, $V_i(t)$, and let us model $V_i(t)$ as the sum of three components: a macroeconomic one $F_i$, influenced by the business cycle; a microeconomic component $M_i$, which accounts for the contagion effects produced by the defaults of the major clients of a firm; and an idiosyncratic random term $\varepsilon_i$.

As in Barro and Basso (2005), the macroeconomic component $F_i(t)$ is described by a factor model
\[ F_i(t) = \sum_{j=1}^{t} \beta_j^{(i)} Y_j(t) \quad t = 0, 1, \ldots, \quad (1) \]

where \( Y(t) = (Y_1(t), Y_2(t), \ldots, Y_J(t)) \) is the vector of the values at time \( t \) of the driving factors, \( s(i) \in \{1, \ldots, S\} \) is the economic sector of firm \( i \), and \( \beta_j^{(i)} \) is the weight of factor \( j \) for the firms of sector \( s \). The driving factors \( Y_j(t) \) are assumed to follow some stochastic process, with covariance matrix \( \sum_y(t) \).

In order to model the microeconomic component \( M_i(t) \), let us define the following measure \( D_i(t) \) of the distress suffered by firm \( i \) at time \( t \), as the difference between the average default rate of the economy \( p(t) \) and the percentage of the turnover of firm \( i \) sold to clients which defaulted at time \( t \)

\[ D_i(t) = p(t) - \left[ \sum_{k \in C_i(t)} \delta_k(t) w_k(t) + p(t) r_i(t) \right], \quad (2) \]

where \( C_i(t) \) denotes the set of the major clients of firm \( i \) at time \( t \), \( w_k \) is the percentage of the sales to major client \( k \) on the turnover of firm \( i \), \( r_i(t) = 1 - \sum_{k \in C_i(t)} w_k(t) \) is the per cent value of the turnover of firm \( i \) sold to all the minor clients and \( \delta_k(t) \) is a binary value which takes value 1 if client \( k \) defaults at time \( t \), 0 otherwise. Notice that the distress measure \( D_i(t) \) has a positive value if the percentage of the turnover of the firm sold to clients which defaulted at time \( t \) is lower than the average default rate in the economy and a negative value if it is higher. The basic idea is that the distress component affects the health of a firm with a one period delay, and its effects decay exponentially in time.

The microeconomic component \( M_i(t) \) can be modelled in the following way

\[ M_i(t) = \mu_{s(i)} \sum_{\tau=1}^{\infty} \lambda_s^\tau D_i(t - \tau) , \quad (3) \]

where \( \mu_s \in \mathbb{R} \) is a real parameter dependent on the economic sector of the firm and \( 0 \leq \lambda_s < 1 \) is the dampening factor which determines the distress memory of the firms in sector \( s \).
The residual idiosyncratic terms $\epsilon_i(t)$ are assumed to be normally distributed with zero mean and standard deviation $\sigma_{i(t)}$, mutually independent and independent of the driving factors $Y_j(t)$.

Therefore, $V_i(t)$ is given by

$$V_i(t) = F_i(t) + M_i(t) + \epsilon_i(t) = \sum_{j=1}^{J} \beta_{j}^{i(t)} Y_j(t) + \mu_{i(t)} \sum_{\tau=1}^{\infty} \lambda_{i(t)}^{\tau} D_{j}(t-\tau) + \epsilon_i(t). \quad (4)$$

To analyze the credit quality upgrades/downgrades of the firms in the portfolio we adopt the commonly used rating approach. Let us consider an ordered set of rating classes $\{1, \ldots, K\}$ that reflect the credit quality of the firms in the portfolio through a mapping $i \mapsto r_i \in \{1, \ldots, K\}$, where $1$ represents the best rating class and $K$ the worst one, $K+1$ representing then the absorbing default state. We assume that the initial classification is determined a priori by some rating system, either external (e.g. provided by an external rating agency as Moody’s or Standard & Poor’s) or internal (when a bank internal rating systems is used).

As it is generally done in the framework of structural models, let us assume that there exists a set of sector-specific thresholds

$$-\infty = d_{j,k+1}^{s} \leq d_{j,k}^{s} \leq \ldots \leq d_{j,1}^{s} \leq d_{j,0}^{s} = +\infty \quad (5)$$

for the proxy $V_i(t)$ of the value of firms in the $k$-th rating class, such that, if $r_i(t) = k$, then $r_i(t+1) = k'$ if and only if $V_i(t+1) \in [d_{k,k'}^{s(i)}, d_{k,k'-1}^{s(i)}]$.

Of course the determination of these thresholds becomes a crucial point in our model. To this aim, let us observe that we may estimate the probability $p_{i,k,k'}^{s(t)}$ of transition in one year from rating $k$ to rating $k'$ from historical data referring to large populations of firms; such estimates, if taken over a sufficiently long period of time, give an estimate of the unconditional rating transition probabilities, since they may be approximately considered as free of cyclical effects connected to the current state of the economy.

For example, let us assume to use the time series of the one-year credit transition matrices for years $1, \ldots, T$. It is known that the arithmetic mean
of the one-year rating transition frequencies gives an estimate of the unconditional rating transition probabilities that underestimates the default probabilities in the best rating classes. In order to avoid such a drawback, following an idea similar to that discussed in Lando and Skødeberg (2002), we could use the following maximum likelihood estimator for Markov chains under a time homogeneity assumption:

\[ P = \exp(\Lambda) \quad \text{with} \quad \Lambda_{i,j} = \frac{N_{i,j}(T)}{\int_0^T N_i(t) \, dt} \quad (6) \]

where \( N_i(t) \) is the number of firms in rating class \( i \) at time \( t \) and \( N_{i,j}(T) \) is the total number of transitions from rating \( i \) to rating \( j \) over the time horizon of interest.

Moreover let us observe that by construction \( E[M_i(t)] = 0 \) for all \( t \), so that the estimate of the unconditional rating transition probabilities, if based on a sufficiently large sample of firms, may also be considered as free of contagion effects. Hence, if eq. (4) has a stationary state, the rating transition matrix (6) gives an estimate of the unconditional transition matrix of the model in this stationary state.

For example, let us assume that the macroeconomic part is described by a single factor model which follows this mean reverting \( AR(1) \) process

\[ Y(t+1) = Y(t) + a(b - Y(t)) + \sigma_y u(t+1) \quad , \quad (7) \]

where \( u(t) \sim N(0,1) \), \( a,b \in R \), \( \sigma_y > 0 \) and let \( \beta = 1 \) for all sectors. Then in the stationary state the macroeconomic component is normally distributed with mean equal to the long term mean \( b \) of \( Y(t) \) and standard deviation \( \sigma_y \).

As for the microeconomic term, we may assume that it is approximately normally distributed with mean 0 and standard deviation \( \sigma_m \), and that it is independent both of the stationary state macroeconomic component, and of the idiosyncratic term.

Under these assumptions, the value of \( V_i(t) \) when the macroeconomic term
in stationary state is normally distributed with mean 0 and standard deviation \( \sqrt{\sigma_y^2 + \sigma_M^2 + \sigma_i^2} \).

In general, let \( G \) denote the probability distribution function of \( V_i(t) \) when the macroeconomic term is in the stationary state; if \( G \) is invertible, then the rating transition thresholds \( d_{k,k'}^{s} \) can be computed as follows

\[
d_{k,k'}^{s} = G^{-1}(p_{k,k+1} + p_{k,k} + \ldots + p_{k,K+1})
\]

where

\[
G(d_{k,k-1}^{s}) - G(d_{k,k}^{s}) = p_{k,k}^{s}.
\]

4. Simulation analysis of rating transitions

In order to test the model proposed in the previous section and study its dynamic behaviour we have carried out a wide simulation analysis by randomly generating a portfolio of bank loans with \( N=10000 \) positions issued to SMEs.

For each firm in the portfolio the number of clients has been randomly generated according to a normal distribution with mean 50 and standard deviation 25, while the volume of sales to each client has been generated according to a lognormal distribution with parameters 5 and 2.

The default or survival status of each client at each time period has been generated according to a Bernoulli random variable with mean equal to the average default rate of the economy \( p(t) \); moreover, each time a major client defaulted, we assumed that in time it is replaced by another client with the same business volume.

The number of rating classes considered is \( K = 7 \), corresponding to the classes from AAA to C in the S&P classification, and each firm was assigned an initial rating class randomly generated according to the S&P rating distribution reported in table 1.
<table>
<thead>
<tr>
<th>Rating class</th>
<th>Relative weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.043730</td>
</tr>
<tr>
<td>AA</td>
<td>0.141370</td>
</tr>
<tr>
<td>A</td>
<td>0.273947</td>
</tr>
<tr>
<td>BBB</td>
<td>0.224433</td>
</tr>
<tr>
<td>BB</td>
<td>0.151319</td>
</tr>
<tr>
<td>B</td>
<td>0.158029</td>
</tr>
<tr>
<td>C</td>
<td>0.007173</td>
</tr>
</tbody>
</table>

Table 1. Relative distribution of obligors in the rating classes AAA - C (source: S&P technical report, 2003).

Once generated the initial composition of the portfolio, in the simulations the defaulted obligors were replaced by new randomly generated obligors, so that the number of positions in the portfolio was kept constant in time. As in eq. (7), for the macroeconomic term (1) we have considered a single factor which follows a mean reverting $AR(1)$ process with $a = 0.5$, $b = 1$ and $\sigma_y = 0.08$. As regards the parameters $\mu$ and $\sigma$, which represent the relative impact of the microeconomic and the idiosyncratic components on $V_i(t)$, respectively, the simulations have been carried out for a set of different values, namely $\mu = 0, 10, 20, 30, 40, 50$ and $\sigma = 0.3, 0.4, 0.5, 0.6, 0.7$.

For the determination of the thresholds $d^*_k,k'$ in the first place we have estimated the one-year rating transition matrix from the time series of S&P historical rating matrices in the period 1988–2002 using the MLE estimator (6); the resulting transition matrix is presented in table 2.
In the second place we have carried out a first set of Monte Carlo simulations in order to analyze the distribution of the microeconomic part $M_i(t)$ for different values of $\mu$ and $\sigma$ and compute the rating thresholds for the different rating classes. The firm specific information about the past (for $t < 0$) were assumed to be not available and accordingly in eq. (3) we set $D_i(t) = 0$ for $t = -1, -2, \ldots$.

For each couple ($\mu$, $\sigma$) we generated 10000 paths for $Y(t)$ and $\varepsilon_i(t)$ on a time horizon of 10 years, with a one-year time step. The empirical results obtained confirms that in each period $M_i(t)$ can be considered as approximately normally distributed with mean 0. As for as the value of the standard deviation $\sigma_M$ is concerned, it turns out to be not only linearly dependent on $\mu$ (which can be immediately seen from eq. (3)), but also approximately linearly dependent on $\sigma$ (see table 3).

Using the values obtained for $\sigma_M$ in this first set of simulations, we have computed the rating transition thresholds $d_{k,k}'$ for each rating class according to eqs. (8)-(9). An example of the thresholds obtained for the different rating classes is presented in table 4. These rating thresholds were held constant over time in the simulations carried out in the second step.
Table 3. Values of $\sigma_M$ for different values of $\mu$ and $\sigma$.

<table>
<thead>
<tr>
<th>$\sigma/\mu$</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.000137</td>
<td>0.000256</td>
<td>0.000386</td>
<td>0.000513</td>
<td>0.000647</td>
</tr>
<tr>
<td>0.4</td>
<td>0.000414</td>
<td>0.000824</td>
<td>0.001238</td>
<td>0.001631</td>
<td>0.002057</td>
</tr>
<tr>
<td>0.5</td>
<td>0.000743</td>
<td>0.001489</td>
<td>0.002239</td>
<td>0.002971</td>
<td>0.003725</td>
</tr>
<tr>
<td>0.6</td>
<td>0.001041</td>
<td>0.002084</td>
<td>0.003143</td>
<td>0.004173</td>
<td>0.005211</td>
</tr>
<tr>
<td>0.7</td>
<td>0.001298</td>
<td>0.002595</td>
<td>0.003865</td>
<td>0.005164</td>
<td>0.006459</td>
</tr>
</tbody>
</table>

Table 4. Rating thresholds of the different rating classes for $\mu = 30$ and $\sigma = 0.3$.

<table>
<thead>
<tr>
<th>Rating Class</th>
<th>Rating Thresholds</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.5548 0.2972 0.13619 0.11575 -0.09914 -0.15469 -0.15469</td>
</tr>
<tr>
<td>AA</td>
<td>1.77638 0.56972 0.23383 0.04936 -0.02166 -0.15469 -0.15469</td>
</tr>
<tr>
<td>A</td>
<td>2.02165 1.68754 0.4887 0.22362 0.07857 -0.04099 -0.09914</td>
</tr>
<tr>
<td>BBB</td>
<td>2.09915 1.87154 1.5095 0.48068 0.25065 0.08474 0.02014</td>
</tr>
<tr>
<td>BB</td>
<td>$+\infty$ 1.94253 1.71998 1.40309 0.55545 0.3254 0.22883</td>
</tr>
<tr>
<td>B</td>
<td>2.15473 1.87985 1.80637 1.69028 1.44175 0.59745 0.4703</td>
</tr>
<tr>
<td>C</td>
<td>1.87154 1.86014 1.74652 1.72464 1.57813 1.40021 0.95894</td>
</tr>
</tbody>
</table>

In the third place, we have carried a second set of simulations in order to study the dynamic behaviour of the model, and to analyze the values of the main quantities of interest as time varies. Again, we generated 10000 paths of the macroeconomic and of the idiosyncratic component for each couple $(\mu, \sigma)$, as in the first step simulations. In this set of simulations we focused our attention on the analysis of the one-year rating transition matrices, the default rate for each rating class and the resulting average default rate of the portfolio, and the distribution of the firms in the portfolio in the different rating classes.

As regards the one-year rating transition matrices, the simulation results indicate that on average it takes three years for the system to get rid of the
initial conditions, while after this initial period the behaviour of these matrices is quite stable in time, in the sense that the average transition matrices, computed by averaging the transition matrices obtained over all the 10000 paths simulated for the macroeconomic factor, do not change significantly as time varies. Two examples of the matrices observed at different times are shown in tables 4 and 5.

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.92575</td>
<td>0.06182</td>
<td>0.00914</td>
<td>0.00054</td>
<td>0.00223</td>
<td>0.00013</td>
<td>0</td>
<td>0.00039</td>
</tr>
<tr>
<td>AA</td>
<td>0.00671</td>
<td>0.90788</td>
<td>0.07751</td>
<td>0.00621</td>
<td>0.00072</td>
<td>0.00056</td>
<td>0</td>
<td>0.00042</td>
</tr>
<tr>
<td>A</td>
<td>0.00056</td>
<td>0.01366</td>
<td>0.93356</td>
<td>0.04486</td>
<td>0.00518</td>
<td>0.00131</td>
<td>0.00028</td>
<td>0.00059</td>
</tr>
<tr>
<td>BBB</td>
<td>0.00023</td>
<td>0.0025</td>
<td>0.04946</td>
<td>0.89804</td>
<td>0.04058</td>
<td>0.00688</td>
<td>0.00095</td>
<td>0.00136</td>
</tr>
<tr>
<td>BB</td>
<td>0</td>
<td>0.0013</td>
<td>0.00951</td>
<td>0.08879</td>
<td>0.82138</td>
<td>0.06241</td>
<td>0.00890</td>
<td>0.00770</td>
</tr>
<tr>
<td>B</td>
<td>0.00011</td>
<td>0.00241</td>
<td>0.00256</td>
<td>0.00884</td>
<td>0.06576</td>
<td>0.81990</td>
<td>0.05394</td>
<td>0.04649</td>
</tr>
<tr>
<td>C</td>
<td>0.00275</td>
<td>0.00033</td>
<td>0.00566</td>
<td>0.00178</td>
<td>0.02212</td>
<td>0.06830</td>
<td>0.45034</td>
<td>0.44873</td>
</tr>
</tbody>
</table>

Table 5. Rating transition matrix obtained with $\mu = 30$ and $\sigma = 0.3$ at time $t=4$.

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.92753</td>
<td>0.06038</td>
<td>0.00883</td>
<td>0.00051</td>
<td>0.00221</td>
<td>0.00015</td>
<td>0</td>
<td>0.00039</td>
</tr>
<tr>
<td>AA</td>
<td>0.00682</td>
<td>0.90852</td>
<td>0.07685</td>
<td>0.00613</td>
<td>0.00071</td>
<td>0.00056</td>
<td>0</td>
<td>0.00041</td>
</tr>
<tr>
<td>A</td>
<td>0.00058</td>
<td>0.01391</td>
<td>0.93377</td>
<td>0.04449</td>
<td>0.0051</td>
<td>0.00131</td>
<td>0.00027</td>
<td>0.00058</td>
</tr>
<tr>
<td>BBB</td>
<td>0.00024</td>
<td>0.00257</td>
<td>0.05011</td>
<td>0.89788</td>
<td>0.04013</td>
<td>0.0068</td>
<td>0.00093</td>
<td>0.00134</td>
</tr>
<tr>
<td>BB</td>
<td>0</td>
<td>0.00136</td>
<td>0.00971</td>
<td>0.08958</td>
<td>0.82094</td>
<td>0.06198</td>
<td>0.00882</td>
<td>0.00761</td>
</tr>
<tr>
<td>B</td>
<td>0.00012</td>
<td>0.00245</td>
<td>0.00261</td>
<td>0.00894</td>
<td>0.06645</td>
<td>0.81976</td>
<td>0.05358</td>
<td>0.04607</td>
</tr>
<tr>
<td>C</td>
<td>0.00279</td>
<td>0.00034</td>
<td>0.00569</td>
<td>0.00187</td>
<td>0.02248</td>
<td>0.069</td>
<td>0.45224</td>
<td>0.44557</td>
</tr>
</tbody>
</table>

Table 6. Rating transition matrix obtained with $\mu = 30$ and $\sigma = 0.3$ at time $t=10$.

We have also measured the distance between the average of the rating transition matrices obtained with the simulation at times $t=3,4,\ldots,10$ and the initial rating transition matrix estimated using the MLE (6) and presented in
As a measure of the distance between two matrices $P$ and $Q$ we used

$$d(P, Q) = \sum_{i,j} |p_{i,j} - q_{i,j}|.$$ 

As can be seen in figures 1 and 2, it turns out that the distance is quite small for the smaller values of $\mu$ while it increases as $\mu$ increases and it is also very sensitive to the increments of $\sigma$. Moreover, it can be observed that when $\mu$ has a strictly positive value the distance increases with $\sigma$, while the converse holds when $\mu$ is equal to 0, i.e. if the microeconomic component is not present in the model. In addition, the simulation results tend to give rating transition matrices with smaller diagonal elements and higher off-diagonal elements than the initial transition matrix, with the effect to increase the probability of changing class (including the default probability for the different rating classes) and to reduce that of staying in the same class.

![Distance between the simulation and the initial transition matrices for different values of $\sigma$](image)

Figure 1: Distance between the average rating transition matrices obtained with the simulation and the initial matrix as $\mu$ varies for different values of $\sigma$. 
As for as the dynamic behaviour of the average default rate of the portfolio is concerned, the simulation outcomes indicate that it tends to converge to a limit value as time increases. For small values of both $\mu$ and $\sigma$, this limit value is very close to the initial average default rate obtained using the estimated initial transition matrix, while it is significantly higher for higher values of $\mu$ and especially of $\sigma$. An example of the dynamic behaviour of the portfolio average default rate is shown in figure 3.

Furthermore, we have analyzed the dynamics of the distribution of the firms in the different rating classes as time varies. The results suggest that the model shows the tendency to increase in time the population of the “central” rating classes A and BBB, and to slightly decrease the others, except for the extreme classes AAA and C, whose relative weight keeps nearly constant. This behaviour can be observed for all pairs of values for $\mu$ and $\sigma$; an example of this tendency is shown in figure 4.
5. Concluding remarks

In this contribution we have proposed a credit contagion model which explicitly takes into account both a macroeconomic effect and a
microeconomic term which describes the counterparty risk. In a structural approach, we have introduced a set of thresholds for the value of a firm whose passage induces either a downgrade or an upgrade of the credit quality of the firm considered and leads to a change in the rating class assigned to it.

The dynamic properties of the model and the effects of the counterparty risk on a portfolio of bank loans have been studied by means of a wide Monte Carlo simulation analysis.

References


