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A credit contagion model for loan portfolios in a network of firms with spatial interaction

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Abstract. This contribution studies the effects of credit contagion on the credit risk of a portfolio of bank loans. To this aim we introduce a model that takes into account the counterparty risk in a network of interdependent firms that describes the presence of business relations among different firms. The location of the firms is simulated with probabilities computed using an entropy spatial interaction model. By means of a wide simulation analysis we use the model proposed to study the effects of default contagion on the loss distribution of a portfolio.

Keywords: credit risk, bank loan portfolios, contagion models, entropy spatial models.

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1 Introduction

The main aim of this contribution is to study the effects of credit contagion on the credit risk of a portfolio of bank loans.

Different approaches have been proposed in the literature in order to analyze the credit risk of a portfolio of bank loans: among others, we may cite for example Aguiar, Forest and Roden [1], Kern and Rudolf [7], Westgaard, van der Wijst [13] and Lucas et al. [8].

Some recent approaches, proposed by Giesecke and Weber [5], Neu and Kühn [9] and Egloff, Leippold and Vanini [4], introduce some models that take into account both the dependence on the business cycle and a direct contagion effect among the firms in the economic system.

In this paper we introduce a model that takes into account the counterparty risk in a network of interdependent firms that describes the presence of business relations among different firms. The model consists of two main components which describe the counterparty risk and the network of the business relations, respectively.

To describe the counterparty risk we use the discrete time model proposed in Barro and Basso [2], that models the asset value of a firm following a structural approach. In particular, the value of a firm is described by the sum of three terms: a macroeconomic component which considers the influence of the business cycle through a factor model, a microeconomic component which models the business connections with other firms and a residual random idiosyncratic term.

The microeconomic component is designed to take into consideration the direct business connections among the firms in the network, so that the default of a firm may cause financial distress, till default, to its suppliers. As a result, a contagion mechanism is introduced in the model.

In order to study the propagation of the defaults in the system and its effects on the risk of a bank loan portfolio, we apply a Monte Carlo simulation technique in order to build a number of proper networks of firms and to simulate the behavior of the system.

A network of firms is simulated by taking into account different features, among which the economic sector and the geographical location. In order to simulate the location of the firms, we introduce an entropy spatial interaction model which considers the distance among the different geographical areas and the economic weight of each area.

A wide simulation analysis is carried out by generating networks that represent loan portfolios of an Italian bank and studying the effects of default contagion on the loss distribution of the portfolio.

The structure of the paper is the following. In Section 2 we briefly review some contributions on the modelling of the counterparty risk in networks of business relations. Section 3 introduces entropy spatial interaction models while Section 4 presents the model used to build the network of interdependent firms with spatial interaction. In Section 5 we discuss the model used to take into account the counterparty risk in the bank loan portfolio. Finally, Section 6 presents the results of an empirical investigation of credit contagion carried out with a Monte Carlo simulation technique.

2 Counterparty risk in business relations networks

Different recent contributions in the literature pointed out the importance of taking into account the counterparty risk, which affects the overall risk of a portfolio of bank loans through a contagion effect.

In particular, some contributions introduce an additional source of risk to allow for a potential interdependence among the positions in the portfolio; along this line, see for example Carling et al. [3].

On the other hand, following a different approach, other contributions try to explicitly model a microeconomic dependence introducing an analytic or stylized descriptions of the business connections among firms.

Giesecke and Weber [5] propose a statistical model of contagion based on the description of local firm interactions. They assume a homogeneous economy in which firms have the same number of business partners and are of equal size. Moreover, they introduce the hypothesis of a symmetric interaction of each firm with its neighbors.

Neu and Khun [9] analyze the counterparty risk in the framework of structural models for loan portfolios. By applying to this context a lattice gas model from physics they use functional description of couplings among counterparties to study the impact of counterparty risk on capital allocation.

In order to account for credit contagion Egloff et al. [4] propose to embed microstructural interdependencies into a macroeconomic factor model. The microeconomic dependencies among firms are modelled through a weighted network in which edges correspond to business relations and the weights on edges accounts for the intensity of such relations.

Schellhorn and Cossin [11] propose a network economy in which the firms are connected by (possibly) looping lending relationships and apply queueing theory to analyze how the default of a firm can influence the default status of the other firms in the network.

The common aim of these contributions is to identify the presence of concentration of risk caused by the dependencies between the firms in the portfolio. The results obtained with these different approaches confirm that the modelling of counterparty risk allows to better describe the loss distribution and the risk profile of a portfolio.

In addition, an empirical analysis carried out by Grunert and Weber [6] on the recovery rates of bank loans in Germany considers among the explanatory variables also some business connection variables such as intensity of the relationships and distance.

3 Entropy spatial interaction models

The network of firms that will be built in next section will consider also a spatial dimension, in the sense that the firms will be assumed to be located in different areas. The localization will be made by resorting to a proper entropy spatial interaction model which takes into account both the economic weight of the different areas considered and their distance.

Entropy models are widely used in urban and regional sciences to study the interaction flows between a set of origins and a set of destinations (see for example Wilson [14]). These flows generally represent sets of agents that move from an origin to a destination

zone; typical examples are the journey-to-work flows of commuting workers, the flows of users towards service centers (hospitals, schools etc.) and the flows of customers towards commercial centers.

Entropy models are included in a wider class of models, called spatial interaction models, that have the property to disperse the agents coming from an origin among all the destinations, instead of assigning all of them to the nearest one.

With respect to the other spatial interaction models, entropy models have the property that they can be obtained as an optimal solution of a mathematical programming problem. In this mathematical program the dispersion of the origin-destination flows is maximized by maximizing the entropy of the system (see Wilson [14]). This link between entropy models and the maximization of the entropy, studied by Wilson since 1970, made these models the most widely used in the class of spatial interaction models.

In particular, these models can be obtained by maximizing the entropy of the system under the available information on the distance matrix between the origins and destinations considered in the network and on the weights assigned to these origins and destinations, which account for the size of the outgoing (for an origin) or incoming (for a destination) flows.

Let:

- $I_m = \{1, \dots, m\}$ be the set of origins;
- $I_n = \{1, \dots, n\}$ be the set of destinations;
- T_{ij} , for $i \in I_m$ and $j \in I_n$ be the the flow of agents from origin i to destination j , generated by the model;
- $O_i = \sum_{j=1}^n T_{ij}$, for $i \in I_m$ be the total flow going out from origin i , observed in the system;
- $D_j = \sum_{i=1}^m T_{ij}$, for $j \in I_n$ be the total flow coming into destination j , observed in the system;
- $T = \sum_{i=1}^m \sum_{j=1}^n T_{ij} = \sum_{i=1}^m O_i = \sum_{j=1}^n D_j$ be the total flow observed;
- d_{ij} , for $i \in I_m$ and $j \in I_n$ be the distance between origin i and destination j ;
- $p_{ij} = T_{ij}/T$, for $i \in I_m$ and $j \in I_n$ be the share of total flow which moves from origin i to destination j ;
- $o_i = O_i/T$, for $i \in I_m$ be the share of total flow going out from origin i ;
- $d_j = D_j/T$, for $j \in I_n$ be the share of total flow coming into destination j .

A spatial interaction model constrained both to origins and destinations generates the origin-destination flows in this way:

$$T_{ij} = A_i B_j O_i D_j f(d_{ij}) \quad i = 1, \dots, m; \quad j = 1, \dots, n \quad (1)$$

where

$$A_i = \left[\sum_{j=1}^n B_j D_j f(d_{ij}) \right]^{-1} \quad i = 1, \dots, m \quad (2)$$

$$B_j = \left[\sum_{i=1}^m A_i O_i f(d_{ij}) \right]^{-1} \quad j = 1, \dots, n \quad (3)$$

and the impedance function f is a decreasing function of distance which makes the flows depend on the distance between the origin and the destination.

In entropy models the impedance function is of exponential type:

$$f(d_{ij}) = e^{-\beta d_{ij}} \quad (4)$$

where $\beta \in \mathbb{R}^+$ is a real parameter.

Alternatively, it is possible to define the entropy model on the shares of total flows p_{ij} rather than on the flows T_{ij} :

$$p_{ij} = a_i b_j o_i d_j f(d_{ij}) \quad i = 1, \dots, m; \quad j = 1, \dots, n \quad (5)$$

where

$$a_i = \left[\sum_{j=1}^n b_j d_j f(d_{ij}) \right]^{-1} \quad i = 1, \dots, m \quad (6)$$

$$b_j = \left[\sum_{i=1}^m a_i o_i f(d_{ij}) \right]^{-1} \quad j = 1, \dots, n \quad (7)$$

The model defined using the variables p_{ij} may be very useful, since these variables can be considered as probabilities: p_{ij} is the probability that an agent in the system will belong to the flow of agents moving from origin i to destination j . It can be proved that if we compute the flows T_{ij} from model (1)–(3) and then divide all of them by total flow T we obtain exactly the same probabilities p_{ij} given by model (5)–(7); viceversa, if we compute the probabilities p_{ij} with model (5)–(7) and then multiply them by T_{ij} we obtain the same flows T_{ij} given by model (1)–(3).

Besides the spatial interaction model constrained both to origins and destinations (5)–(7), which is also called a doubly constrained models, it is possible to define a spatial interaction model which is constrained only to either origins or destinations.

For example, in the entropy model constrained to destinations the shares of total flows p_{ij} are computed in the following way

$$\begin{aligned} p_{ij} &= d_j \frac{e^{-\beta d_{ij}}}{\sum_{j=1}^n e^{-\beta d_{ij}}} = \\ &= b_j d_j e^{-\beta d_{ij}} \quad i = 1, \dots, m; \quad j = 1, \dots, n, \end{aligned} \quad (8)$$

where

$$b_j = \left[\sum_{i=1}^m e^{-\beta d_{ij}} \right]^{-1} \quad j = 1, \dots, n. \quad (9)$$

It can be proved that this model is obtained by imposing the constraint that the sum of flows that come into a destination coincides with the share of total flow of this destination, i.e.

$$\sum_{i=1}^m p_{ij} = d_j \quad j = 1, \dots, n. \quad (10)$$

In next section we will use a special entropy model constrained to destinations in which the total flow coming out of origin i is set equal to 1, for all origins $i = 1, 2, \dots, m$. In this way the probabilities p_{ij} are scaled in such a way as they sum to 1 for each row of the matrix (p_{ij}) of the relative flows

$$\sum_{i=1}^n p_{ij} = o_i = 1 \quad i = 1, \dots, m, \quad (11)$$

so that they represent conditional probabilities, in which the origin is assumed to be known while the destination has to be randomly generated with probability p_{ij} .

4 Modelling a network of interdependent firms with spatial interaction

The main aim of this contribution is to study the effects of credit contagion on the credit risk of a portfolio of bank loans. To this aim, we introduce a model that takes into account the counterparty risk in a network of interdependent firms in which the spatial diffusion of the business relations is described by an entropy spatial interaction model.

The model consists of two main components which describe the counterparty risk and the network of the business relations, respectively. In this section we present the model used to build the business relations network while the model used to describe the counterparty risk is presented in next section. Afterwards, the two models will be used combined to study the effects of credit contagion in section 6.

Formally, the business connections are modelled using a weighted network. In particular, let us consider a weighted network in which the nodes represent the firms, directed edges connect each firm with its major clients and the weight associated to each edge is given by the percentage of sales to this client on the turnover of the firm.

The firms included in the network are given by the N firms of the bank portfolio under consideration and, in addition, by the firms that are major clients of any of the firms in the portfolio. Two examples of networks of business connections are shown in figures 1 and 2; in these figures, the firms in the bank portfolio are represented by the nodes inside the ellipse, while the arcs directed to points outside the ellipse represents the business connections with “major” clients that are not in the bank portfolio.

Figure 1: Example of network of business connections for $q > 0$.

The basic idea is that if a client which represents a significant percentage of the turnover of firm i , let us say above a given threshold θ , defaults, this may result in a serious cause of distress for firm i , while if a “minor” client, with purchases below threshold θ , defaults the repercussions may be negligible. Hence, we may take into consideration in the network only the connections with the clients with a percentage of turnover above a given threshold θ .

In order to study the propagation of the defaults in the system and the effects on the risk of a bank loan portfolio, we have simulated with a Monte Carlo simulation technique a number of networks of firms and simulated the behavior of the system on each of these networks.

These networks of firms are built by taking into consideration also a spatial dimension and the firms are located in different areas by resorting to an entropy spatial interaction model which takes into account both the economic weight of the different areas considered and their distance.

In detail, the bank loan portfolios are generated by fixing the number of financial positions (1 000, in the empirical analysis carried out) and for each position in the portfolio the location of the firm is randomly simulated using a special entropy spatial interaction model constrained to destinations in which the flows represent conditional probabilities.

More precisely, let us denote by 1 the area in which the bank analyzed is situated, by π_{1j} the probability that a firm in the loan portfolio of the bank is located in area j , with $j = 1, 2, \dots, n_a$, by d_{1j} the distance between the area 1 (of the bank) and area j and by W_j the weight assigned to the destination area j , representing the relative attractiveness of the area computed on the ground of its economic importance; in the spatial interaction

Figure 2: Example of network of business connections for $q = 0$.

model the weights W_j of the destination areas represent the share of total flow coming into destination j .

Using an entropy model constrained to destinations, the probability π_{1j} that a firm in the loan portfolio of a bank situated in area 1 is located in area j is computed as follows

$$\pi_{1j} = W_j \frac{e^{-\beta d_{1j}}}{\sum_{j=1}^n W_j e^{-\beta d_{1j}}} \quad j = 1, 2, \dots, n_a. \quad (12)$$

It can be easily seen that in this model the probabilities π_{ij} are scaled in such a way as to sum to 1 for each row of the probability matrix (π_{ij})

$$\sum_{i=1}^n \pi_{ij} = 1 \quad i = 1, \dots, m, \quad (13)$$

so that they represent conditional probabilities (actually, we only need to compute the first row of this matrix). In this way the destination (the location of the firms in the bank portfolio) can be randomly generated with probabilities π_{1j} . This constraint accounts for the fact that the origin is known and entails that the total flow coming out of origin i is set equal to 1, for all origins $i = 1, 2, \dots, m$ for which we need to compute the probabilities (actually, only the first origin).

Besides the area in which it is located, for each firm in the bank loan portfolio the model takes into account also its economic sector. In the simulations carried out, the information on the economic sectors are randomly generated using the widely used Global Industry

Classification Standard (GICS) classification, with probabilities computed by properly aggregating and scaling the input-output table of the Italian economy. The same probabilities are also used to randomly generate the economic sector of each “major” client of each firm in the portfolio.

Once generated the firms in the bank loan portfolio, we need to randomly generate the business connections among these firms, i.e. the directed arcs (f_i, f_j) connecting the nodes of the network and the weights associated to these arcs, given by the percentage of sales to client f_j on the turnover of firm f_i .

This is done by assigning a probability q that a “major” client of a firm belongs to the bank portfolio and randomly generating the belonging to the bank portfolio with such a probability. The value of q greatly influence the structure of the business network: the higher the value of q , the more dense the network will be, the lower the value of q , the more sparse the network will come out. In the special case of $q = 0$, no arc can connect two firms in the same bank loan portfolio; this situation is depicted in figure 2, which refers to the case $q = 0$ and can be compared to the example of figure 1 which illustrates the case $q > 0$.

In detail, we simulate the turnover sold to each client of each firm in the bank loan portfolio and, for the major clients, we randomly generate their economic sector and their belonging to the bank portfolio. Once simulated in such a way the presence of an arc connecting two firms of the portfolio network, the final node of the arc is randomly chosen among the firms in the portfolio which pertain to the sector of the “major” client considered, with probabilities proportional to the turnover of each firm.

5 Modelling counterparty risk in a portfolio of bank loans

Let us now present the model used to describe the counterparty risk. To describe the counterparty risk we use the discrete time model proposed in Barro and Basso [2]. This model focuses on the asset value of firms and, as in a structural approach, a firm defaults when the value of its asset falls below a given threshold.

The asset value of a firm is modelled as the sum of three components: a macroeconomic component F , modelled using a factor model which takes into account the influence of the business cycle; a microeconomic component M which introduces a contagion effect due to the business connections with other firms; a residual idiosyncratic term ε of random nature.

Let us consider a portfolio of bank loans made up of N firms pertaining to S economic sectors and let us denote by $s(i)$ the sector of firm i , with $i = 1, 2, \dots, N$. The asset value of firm i at time t , $V_i(t)$, is defined as follows

$$V_i(t) = F_i(t) + M_i(t) + \varepsilon_i(t) \quad i = 1, 2, \dots, N; \quad t = 0, 1, \dots \quad (14)$$

The value of the macroeconomic component $F_i(t)$ is described by a factor model as follows:

$$F_i(t) = \sum_{j=1}^J \beta_j^{s(i)} Y_j(t) \quad t = 0, 1, \dots, \quad (15)$$

where $Y(t) = (Y_1(t), Y_2(t), \dots, Y_J(t))$ is the vector of the values at time t of the driving factors and β_j^s is the weight of factor j for the firms of sector s (on factor models see for example Schönbucher [12] and Saunders, Xiouros and Zenios [10]).

Each driving factor Y_j , with $j \in \{1, 2, \dots, J\}$, is assumed to follow a stochastic process $\{Y_j(t), t \in \mathbb{N}\}$ which may be specified in different ways. For example, some of the factors could be modelled using normal random variables while for others a mean-reverting process or a different stochastic processes could be more adequate.

The microeconomic component $M_i(t)$ takes into consideration the direct business connections among the firms represented in the weighted network defined in the previous section, so that the default of a firm may cause financial distress, till default, to its suppliers.

As we have seen in the previous section, the business connections taken into consideration are the relations of a firm with its “important” clients, where the importance of client k for firm i is measured by the percentage w_{ik} of the sales to client k on the turnover of the firm i .

Moreover, we assume that the default of a “major” client affects the health of a firm with a one-period delay and that the effects are dampened according to an exponential decay in time. As a result, a contagion mechanism is introduced in the model.

More precisely, the microeconomic component $M_i(t)$ is modelled as a firm-specific term which depends on the distress undergone by firm i due to the defaults observed among its clients in the previous periods. In particular, let us define the distress measure $D_i(t)$ connected to the defaults observed at time t among the clients of firm i , compared to the average default rate observed in the economy at time t , $p(t)$, as follows

$$D_i(t) = p(t) - \left[\sum_{k \in C_i(t)} \delta_k(t) w_{ik}(t) + p(t) r_i(t) \right], \quad (16)$$

where $C_i(t)$ denotes the set of the major clients of firm i ,

$$r_i(t) = 1 - \sum_{k \in C_i(t)} w_{ik}(t) \quad (17)$$

is the per cent value of the turnover of firm i sold to all the minor clients, i.e. the clients of firm i below the threshold θ , and $\delta_k(t)$ is defined as

$$\delta_k(t) = \begin{cases} 1 & \text{if client } k \text{ defaults at time } t \\ 0 & \text{otherwise.} \end{cases} \quad (18)$$

In equation (16) only the defaults of the major clients are taken into consideration explicitly, while for the residual part of the turnover, due to a large number of minor clients, a per cent amount equal to the average default rate $p(t)$ is assumed to default.

The distress measure $D_i(t)$ of firm i at time t is computed as the difference between the average default rate $p(t)$ and the percentage of turnover of firm i sold to clients which defaulted in the period t . Hence, the distress measure $D_i(t)$ is a real number which is negative if at time t the per cent amount of the turnover of firm i due to clients that defaulted in the period t is higher than the average default rate of the economy $p(t)$, while

it is null if it is equal to the average default rate and it has a positive value if it is lower than the average default rate of the economy.

We assume that all the distresses undergone by the firm in the past periods affect the current health of the firm, with a dampening factor which entails an exponential decay in time of the influence of the past defaults. The overall distress influencing the health of firm i at time t is therefore measured as the sum of the effects of all the past defaults of some of its clients as follows

$$\sum_{\tau=1}^{\infty} \lambda_{s(i)}^{\tau} D_i(t-\tau) = \sum_{\tau=1}^{\infty} \lambda_{s(i)}^{\tau} \left[p(t-\tau) - \left(\sum_{k \in C_i(t-\tau)} \delta_k(t-\tau) w_{ik}(t-\tau) + p(t-\tau) r_i(t-\tau) \right) \right], \quad (19)$$

where the parameter λ_s , with $0 \leq \lambda_s < 1$ is the dampening factor which determines the distress memory of firms in sector s . We may noticed that if, as usual, the value of λ_s is sufficiently small, only the first terms in the infinite summation in equation (19) have a non negligible value.

The microeconomic component, $M_i(t)$, takes into account the effects of the past distresses on the health of firm i and is defined as follows

$$\begin{aligned} M_i(t) &= \mu_{s(i)} \sum_{\tau=1}^{\infty} \lambda_{s(i)}^{\tau} D_i(t-\tau) = \\ &= \mu_{s(i)} \sum_{\tau=1}^{\infty} \lambda_{s(i)}^{\tau} \left[p(t-\tau) - \left(\sum_{k \in C_i(t-\tau)} \delta_k(t-\tau) w_{ik}(t-\tau) + p(t-\tau) r_i(t-\tau) \right) \right], \end{aligned} \quad (20)$$

where $\mu_s \in \mathbb{R}_+$ is a real parameter, possibly dependent on the economic sector of the firm. As it can be seen, the microeconomic component $M_i(t)$ is a firm-specific additive term which brings about a rise or a decrease in the value of firm i with respect to macroeconomic component $F_i(t)$, according to the fact that the overall financial distress due to the past defaults of clients is lower or higher than the average distress undergone by the sector of the firm.

In addition to the macro and microeconomic components we have a residual idiosyncratic term $\varepsilon_i(t)$, which is assumed to be normally distributed with zero mean and standard deviation σ_{ε_i} . Moreover, we assume that the residual idiosyncratic terms $\varepsilon_1(t), \varepsilon_2(t), \dots, \varepsilon_N(t)$ are both mutually independent and independent of the factors $Y_1, Y_2, \dots, Y_J$.

Therefore, the asset value of firm i , $V_i(t)$, is defined as the sum of the macroeconomic, microeconomic and residual terms, as follows

$$\begin{aligned} V_i(t) &= F_i(t) + M_i(t) + \varepsilon_i(t) = \\ &= \sum_{j=1}^J \beta_j^{s(i)} Y_j(t) + \mu_{s(i)} \sum_{\tau=1}^{\infty} \lambda_{s(i)}^{\tau} D_i(t-\tau) + \varepsilon_i(t). \end{aligned} \quad (21)$$

6 A simulation analysis of credit contagion

In order to study the effects of default contagion on a portfolio of bank loans we have carried out a Monte Carlo simulation analysis considering different values of the parameters in the model. We have considered a 10-year time horizon.

The number of firms in the portfolios generated is set equal to $N = 1000$, while the number of clients of each firm and the volume of sales for each client are randomly generated according to a normal distribution with mean 50 and standard deviation 25 and to a lognormal distribution with parameters $(5, 2)$, respectively.

For the macroeconomic component we consider one factor simulated according to a mean-reverting process with drift 0.5, volatility 0.3 and long-run mean 1.

We consider $S = 10$ sectors according to the Global Industry Classification Standard (GICS) developed by Morgan Stanley Capital International (MSCI) and Standard & Poor's (S&P). The GICS classification is presented in table 1.

	Sectors
1	Energy Sector
2	Materials Sector
3	Industrials Sector
4	Consumer Discretionary Sector
5	Consumer Staples Sector
6	Health Care Sector
7	Financials Sector
8	Information Technology Sector
9	Telecommunications Services Sector
10	Utilities Sector

Table 1: GICS Sector Classification.

The GICS methodology has been widely accepted as an industry analysis framework for investment research, portfolio management and asset allocation. For a more detailed description of the sectors and of the classification methodology we refer to the documentation which is available at the MSCI web site (www.msci.com/equity/gics.html).

In order to determine the sector of each firm and of each major client of the firms in the portfolio we use an Input/Output table which quantify the relationships between different sectors.

We consider the Input/Output table for the Italian economy for the year 2001 and aggregate it according to the GICS classification. The resulting table is normalized in such a way that the generic element of the matrix a_{ij} gives the probability that a firm in sector i has a client in sector j . Therefore, each row of the table represents a vector of probabilities describing the relations among the sector considered and all the sectors in the economy. In table 2 we present the normalized table used in the simulation study.

In the simulation we consider a bank, which we assume to be located in area 1, and a portfolio of loans. For each obligor in the portfolio one of the feature considered is its geographical location.

Sectors	1	2	3	4	5	6	7	8	9	10
1	0.4610	0.0622	0.1409	0.0890	0.0137	0.0075	0.0056	0.0034	0.0080	0.2086
2	0.0013	0.4092	0.3166	0.1717	0.0357	0.0435	0.0102	0.0037	0.0031	0.0049
3	0.0027	0.1137	0.4836	0.1964	0.0530	0.0286	0.0764	0.0152	0.0178	0.0126
4	0.0054	0.1424	0.1931	0.5121	0.0939	0.0276	0.0121	0.0035	0.0035	0.0066
5	0.0001	0.0342	0.0962	0.2559	0.5596	0.0324	0.0181	0.0007	0.0023	0.0005
6	0.0001	0.0041	0.0172	0.0096	0.0059	0.9579	0.0046	0.0003	0.0002	0.0001
7	0.0024	0.0718	0.2709	0.2543	0.0535	0.0174	0.2969	0.0149	0.0138	0.0041
8	0.0014	0.0636	0.2376	0.1697	0.0256	0.0111	0.2997	0.1291	0.0471	0.0151
9	0.0010	0.0780	0.4214	0.2011	0.0435	0.0355	0.1826	0.0183	0.0058	0.0128
10	0.0063	0.2734	0.1298	0.2103	0.0887	0.0249	0.0225	0.0047	0.0136	0.2257

Table 2: Normalized Input/Output table.

In more detail, we consider 15 different areas: the areas which surround the area of the bank are smaller while the areas which are more far-away are wider. This allows to obtain a higher degree of detail in the classification of the loans in the portfolio.

We assume that the first area, that is the area of the bank, corresponds to the province of Venezia, areas from 2 to 5 covers the Veneto region while areas from 6 to 10 correspond to the other regions in the North of Italy. The remaining areas from 10 to 14 cover the Central and South part of Italy, while area 15 is a wide generic area which includes all the foreign countries.

To determine the area of each firm we apply the entropy spatial interaction model described in section 4.

As for the weights associated to each destination in the entropy spatial interaction model we assume that the population of an area represents an adequate proxy for the economic relevance of the area. For the foreign countries area the economic weight is computed proportionally to the relative weight of the exports over the Italian GNP. The weights are then normalized in such a way as they sum to one.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	41	46	70	120	167	165	159	284	415	265	567	803	1540	1600
2		0	70	98	145	146	166	176	296	426	283	565	770	1453	1600
3			0	43	90	139	186	122	247	378	228	530	765	1503	1600
4				0	54	151	206	155	214	345	261	563	799	1536	1600
5					0	100	256	141	164	295	247	549	790	1534	1600
6						0	279	233	218	349	339	641	882	1626	1600
7							0	308	420	551	414	715	951	1689	1600
8								0	218	338	106	408	660	1415	1600
9									0	139	324	626	878	1633	1600
10										0	442	702	972	1743	1600
11											0	302	600	1345	1600
12												0	367	1043	1600
13													0	774	1600
14														0	1600
15															0

Table 3: Matrix of the distances among the 15 geographical areas considered.

Table 3 presents the distance matrix used in the simulation. Distances are expressed in kilometers and are computed with reference to the most representative city in each area, with the exception of the distances to the foreign countries area 15, which have been chosen roughly, in a subjective manner.

Table 4 describes the different areas considered, the weights W_j assigned to each area, and the probabilities π_{1j} obtained with the entropy spatial interaction model with parameter $\beta = 0.08$.

	Area	W_j	π_{1j} (%)
1	Venezia	0.01096	92.16148
2	Treviso-Belluno	0.01372	4.34255
3	Padova-Rovigo	0.01486	3.15215
4	Vicenza	0.01079	0.33569
5	Verona	0.01115	0.00635
6	Trentino Alto Adige	0.01258	0.00017
7	Friuli Venezia Giulia	0.01624	0.00025
8	Emilia Romagna	0.05399	0.00136
9	Lombardia	0.12265	0.00000
10	North-West Italy	0.08301	0.00000
11	Central Italy 1	0.07939	0.00000
12	Central Italy 2	0.09339	0.00000
13	South Italy	0.17192	0.00000
14	Sicilia-Sardegna	0.09260	0.00000
15	Foreign countries	0.12340	0.00000

Table 4: The geographical areas, the economic weights associated to each area and the probabilities obtained applying the entropy spatial interaction model with $\beta = 0.08$, considered in the simulation study.

Moreover for the microeconomic component we choose a value of $\lambda = 0.15$ for the dampening factor and thus the infinite summation in the microeconomic component is truncated after three terms. This means that the effects of the defaults of major clients persist for no more than three periods.

To initialize the model we assume that no firm specific information is available for the past and thus the lagged terms, before the initial time $t = 0$, have been set equal to 0.

The idiosyncratic term $\varepsilon_i(t)$ is generated according to a normal random variable $N(0, \sigma)$.

In the simulations the default barriers are set equal to zero for all firms, the exposure of the bank with each obligor is held constant and all portfolio losses are measured as a percentage of the overall exposure of the bank portfolio. The recovery rate is set equal to 50% and held constant.

In the simulation study we have considered different values for the parameter q which determines the number of arcs in the network directed to firms in the bank portfolio, that is the number of major clients which are firms in the portfolio of the bank. For each value of q we have analyzed the behavior of the portfolios for different values of μ and σ which determine the relative impact, on the value of a firm, of the microeconomic component and of the residual term, respectively.

The quantities analyzed in the simulation study are the expected and unexpected losses (EL and UL), measured as a percentage of the overall exposure, the VaR and the CVaR with a 99% confidence level.

To analyze EL and UL for each value of the parameter q we have generated 100 different portfolios and for each portfolio we have carried out 1000 simulations for each pair of

parameters μ and σ considered.

However, in order to study the behavior of VaR and CVaR a more detailed description of the tail of the loss distribution is needed, and this cannot be obtained with only 1 000 simulations for each different portfolio and pair (μ, σ) . Therefore, we have limited the analysis of VaR and CVaR to one portfolio and we have generated 100 000 paths for each pair of parameters μ and σ considered.

Tables 5 and figure 3 report the expected losses obtained in the fourth year for different values of q and pairs of parameters (μ, σ) . While tables 6 and figure 4 show the unexpected losses obtained in the fourth year for the same values of q and pairs of parameters (μ, σ) .

In tables 7 and 8 we report the 99 % confidence level VaR and CVaR in the fourth year for the different values of the parameters considered. These results are shown also in figure 5 and 6, respectively.

As can be seen from figure 3–6 the EL, UL, VaR and CVaR increase as the parameter q increases and this is due to the presence of a contagion effect introduced by the microeconomic component. Moreover, as we can expect, all the monitored quantities increase as μ and σ increase.

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μ	σ			
	0.05	0.1	0.15	0.2
$q = 0$				
0	0.00004	0.00107	0.00136	0.00703
0.1	0.00061	0.00289	0.00508	0.00891
0.5	0.00072	0.00398	0.00924	0.01818
1	0.00125	0.00824	0.00908	0.02041
$q = 0.05$				
0	0.00177	0.00510	0.01355	0.02673
0.1	0.00189	0.00587	0.01490	0.02895
0.5	0.00394	0.01010	0.02205	0.03823
1	0.00886	0.01805	0.03419	0.05314
$q = 0.1$				
0	0.00879	0.00501	0.01359	0.02683
0.1	0.00906	0.00578	0.01486	0.02881
0.5	0.00998	0.01009	0.01838	0.03926
1	0.01973	0.01802	0.03389	0.05439
$q = 0.25$				
0	0.00727	0.00802	0.01042	0.01108
0.1	0.00809	0.00883	0.01473	0.02913
0.5	0.00825	0.01022	0.02196	0.03817
1	0.00942	0.00980	0.03438	0.05285
$q = 0.5$				
0	0.00875	0.00901	0.01351	0.02675
0.1	0.00909	0.00991	0.01508	0.03369
0.5	0.00914	0.01011	0.02183	0.03856
1	0.01223	0.01813	0.03360	0.05311
$q = 1.0$				
0	0.02171	0.03849	0.06359	0.07662
0.1	0.03919	0.04584	0.07473	0.07291
0.5	0.05594	0.09912	0.10090	0.12822
1	0.10924	0.11797	0.12443	0.14328

Table 5: Average expected loss on the overall exposure for the simulated bank portfolios in the 4th year for different values of μ , σ and q .

μ	σ			
	0.05	0.1	0.15	0.2
$q = 0$				
0	0.00042	0.00051	0.00091	0.00135
0.1	0.00144	0.00064	0.00105	0.00148
0.5	0.00195	0.00120	0.00173	0.00214
1	0.00236	0.00250	0.00300	0.00326
$q = 0.05$				
0	0.00039	0.00058	0.00082	0.00113
0.1	0.00052	0.00083	0.00102	0.00269
0.5	0.00104	0.00125	0.00193	0.00396
1	0.00277	0.00348	0.00375	0.00479
$q = 0.1$				
0	0.00131	0.00152	0.00091	0.00134
0.1	0.00134	0.00162	0.00202	0.00248
0.5	0.00271	0.00276	0.00367	0.00397
1	0.00306	0.00347	0.00418	0.00531
$q = 0.25$				
0	0.00139	0.00148	0.00182	0.00193
0.1	0.00152	0.00143	0.00189	0.00165
0.5	0.00164	0.00165	0.00193	0.00286
1	0.00237	0.00378	0.00395	0.00417
$q = 0.5$				
0	0.00143	0.00138	0.00188	0.00293
0.1	0.00151	0.00183	0.00293	0.00365
0.5	0.00160	0.00175	0.00335	0.00486
1	0.00226	0.00358	0.00405	0.00617
$q = 1.0$				
0	0.00243	0.00438	0.00588	0.00693
0.1	0.00351	0.00583	0.00793	0.00865
0.5	0.00416	0.00635	0.00855	0.00986
1	0.00626	0.00658	0.00985	0.01117

Table 6: Average unexpected loss on the overall exposure for the simulated bank portfolios in the 4th year for different values of μ , σ and q .

μ	σ			
	0.05	0.1	0.15	0.2
q = 0				
0	0.00101	0.00131	0.00696	0.02497
0.1	0.00159	0.00376	0.00792	0.02668
0.5	0.00191	0.00496	0.01348	0.02678
1	0.00289	0.00935	0.01201	0.04327
q = 0.05				
0	0.00194	0.00678	0.01541	0.03247
0.1	0.00289	0.00766	0.01890	0.03606
0.5	0.00488	0.01368	0.02891	0.04341
1	0.00974	0.02010	0.04506	0.06879
q = 0.1				
0	0.00969	0.00689	0.01569	0.03264
0.1	0.01120	0.00765	0.01987	0.03557
0.5	0.01137	0.01956	0.02435	0.04788
1	0.02035	0.02354	0.04167	0.07645
q = 0.25				
0	0.01123	0.01199	0.02359	0.01456
0.1	0.01274	0.01254	0.02879	0.03245
0.5	0.01458	0.01569	0.02465	0.04567
1	0.01677	0.01598	0.03765	0.06566
q = 0.5				
0	0.01235	0.01258	0.01404	0.02988
0.1	0.01356	0.01443	0.0160	0.03588
0.5	0.01789	0.01799	0.02288	0.04345
1	0.01978	0.02033	0.03561	0.06307
q = 1.0				
0	0.03567	0.04528	0.07685	0.08972
0.1	0.04358	0.06686	0.07877	0.08823
0.5	0.05789	0.12304	0.17890	0.18457
1	0.12076	0.14560	0.18003	0.18321

Table 7: Average level of VaR with a 99 % confidence level for the simulated bank portfolios in the 4th year for different values of μ , σ and q .

μ	σ			
	0.05	0.1	0.15	0.2
q = 0				
0	0.01233	0.01144	0.01203	0.02506
0.1	0.01278	0.01345	0.01376	0.02881
0.5	0.01341	0.01408	0.01713	0.03231
1	0.01321	0.01481	0.01876	0.04993
q = 0.05				
0	0.01254	0.01203	0.01635	0.03288
0.1	0.01308	0.01136	0.01979	0.03781
0.5	0.01347	0.01456	0.03002	0.04435
1	0.01458	0.02134	0.04789	0.07342
q = 0.1				
0	0.01333	0.01456	0.02165	0.04357
0.1	0.01467	0.01601	0.02349	0.04576
0.5	0.01478	0.02001	0.03298	0.06682
1	0.02101	0.02398	0.04609	0.08961
q = 0.25				
0	0.01445	0.01567	0.02576	0.04187
0.1	0.01356	0.01783	0.03098	0.04005
0.5	0.01499	0.01796	0.03101	0.04823
1	0.02098	0.02076	0.04076	0.08854
q = 0.5				
0	0.01667	0.01599	0.01602	0.03345
0.1	0.01456	0.01645	0.01873	0.03881
0.5	0.01832	0.01934	0.02466	0.04465
1	0.02032	0.02376	0.03798	0.08353
q = 1.0				
0	0.04567	0.05487	0.08797	0.09871
0.1	0.04654	0.07645	0.08949	0.09834
0.5	0.06213	0.13548	0.18752	0.20458
1	0.13452	0.01567	0.19978	0.21456

Table 8: Average level of CVaR with a 99 % confidence level for the simulated bank portfolios in the 4th year for different values of μ , σ and q .

Figure 3: Average expected loss on the overall exposure for the simulated bank portfolios in the 4th year for different values of μ , σ and q .

Figure 4: Average unexpected loss on the overall exposure for the simulated bank portfolios in the 4th year for different values of μ , σ and q .

Figure 5: Average level of VaR with a 99 % confidence level for the simulated bank portfolios in the 4th year for different values of μ , σ and q .

Figure 6: Average level of CVaR with a 99 % confidence level for the simulated bank portfolios in the 4th year for different values of μ , σ and q .

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