Monopolistic Competition: A Dual Approach

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Abstract

We study monopolistic competition with preferences characterized by separable indirect utility rather than separable direct utility as in the Dixit-Stiglitz model, with the CES case as the only common case. Other examples lead to exponential or linear direct demand functions. We find that an increase of the number of consumers never affects prices and firms' size, but increases proportionally the number of firms, creating always pure gains from variety. Contrary to the Dixit-Stiglitz model, an increase in individual income increases prices (and more than proportionally the number of varieties) and reduces firms' size if and only if the price elasticity of demand is increasing. We extend the model to and outside good, heterogenous consumers and heterogenous firms *à la* Melitz. Finally, we provide an application to international trade generating pricing to market in a generalized Krugman model.

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The Dixit and Stiglitz (D-S, 1977) model of monopolistic competition and endogenous entry *à la* Chamberlin (1933) has been widely applied in the modern theories of trade (Krugman, 1980; Melitz, 2003) and various fields of macroeconomics. Due to its analytical tractability, most applications of the D-S setting rely on the particular case of constant elasticity of substitution (CES) preferences, which generates equilibrium prices and output levels that are independent both from the number of consumers (the market "size") and the individual income and pure gains from variety associated with larger markets.

Since Krugman (1979), the original D-S model based on non-homothetic preferences has been used to justify markups changing with the market size, but such a general model delivers ambiguous results, with prices that could increase or decrease with the number of consumers (for a recent discussion see Zhelobodko *et al.*, 2012, Dhingra and Morrow, 2012, and Bertoletti and Epifani, 2012): this runs against the empirical evidence for which market size hardly affects mark ups in monopolistically competitive markets (Simonovska, 2010) but reduces them in concentrated markets where entry intensifies competition - for instance see Manuszak (2002) and Campbell and Hopenhayn (2005). Most important, the original D-S model with non-homothetic preferences delivers neutrality of income on prices under free entry, which makes it impossible to rationalize one of the most robust empirical facts in international trade, the existence of pricing to market for tradable goods: for recent evidence on the positive elasticity of prices to income see Alexandria and Kaboski (2011) and Simonovska (2010), who stress the role of more rigid demand in richer countries.[2]

In this work, we develop an alternative model of monopolistic competition which generalizes the CES model but generates more realistic predictions for the impact of changes in population and per capita income. The model is alternative because it is based on a new class of preferences compared to the original D-S model. As well known, the latter assumes "direct additivity": consumers' preferences can be represented by an additively separable utility function. We adopt the assumption of "*in*direct additivity": consumers' preferences can be represented by an additively separable *indirect* utility function. This amounts to assume that the relative demand of two goods does not depend on the price of other goods, and it is different from the assumption of direct additivity, for which the marginal rate of substitution between any two goods does not depend on the consumption of other goods. Most important, the case of CES preferences is the only common ground of these two forms of separability. Therefore our analysis applies to an (almost) entirely different class of well-behaved preferences than D-S. It is also worth noticing that preferences which satisfy indirect additivity generate analytically tractable *direct* demand functions: examples include isoelastic, exponential and linear demands, for which we can also reconstruct the original direct utilities.

Our results are rather different from those received by the literature. First,

[2] For recent alternative analysis of monopolistic competition and trade with non-homothetic preferences able to generate pricing to market see Fajgelbaum *et al.* (2011) adopting a Logit demand, Simonovska (2010) adopting a hierarchic demand and Fieler (2011) studying a two-sector model with CES preferences.

indirect additivity implies that an increase in the number of consumers never affects equilibrium prices and firm size, and just expands linearly the number of goods produced, so as to generate pure welfare gains from variety. Accordingly, these crucial properties of the CES case extend to our wide class of (non-homothetic) preferences. This result is reassuring for the many trade models based on CES preferences (e.g., Krugman, 1980 and Melitz, 2003), and in striking contrast with the general D-S setting, whereby the competitive effects of market size, and even the welfare gains, depend ambiguously on the properties of what Zhelobodko *et al.* (2012) call the "relative love for variety" embedded into preferences. Of course, in the presence of a small number of firms, standard strategic interactions *à la* Bertrand or Cournot would generate the competition effect of an increase in market size that appear to be supported in the industrial organization literature (Manuszak, 2002; Campbell and Hopenhayn, 2005; Etro, 2014).

Second, in our setting preferences directly determine demand elasticity and naturally allow income changes to affect it. If, as natural, the demand elasticity is increasing in the price, a rise of individual income makes demand more rigid, inducing higher prices, a reduction of firm size and a more than proportional increase in the free-entry equilibrium number of firms. This is again in contrast with the result of the D-S setting, whereby equilibrium prices are independent from income, whose growth only expands proportionally the number of varieties, under both CES and non-CES preferences. That "neutrality" simplifies a lot the macroeconomic analysis of technological progress/shocks in applied work (for a recent application see Bilbiie *et al.*, 2012), but it should not be expected to hold in general. Our non-homothetic preferences generate therefore the classic rationale for pricing to market: markups of identical goods will be higher in richer markets because demand is more rigid, a result that is widely supported by the empirical evidence (Alexandria and Kaboski, 2011; Fieler, 2011). The only work we are aware of, that investigates international pricing of identical goods and controls separately for income effects and market size effects is Simonovska (2010), who finds a elasticity of prices to country per capita GDP between 5% and 11% and does not find a significant impact of population on prices: this is in line with our theoretical findings.[3]

We confirm our main results after extending the model to a two-sector economy and introducing heterogeneity of consumers in both income and preferences, we analyze the welfare properties of the decentralized equilibrium and we investigate the role of cost heterogeneity between firms *à la* Melitz (2003) emphasizing the distinct role of changes in population and productivity on the market structure. Finally, we apply our baseline framework to the analysis of international trade between two countries. Initially we focus on costless trade between two countries different in size and income (productivity), which gen-

[3] An additional implication of our model is about the impact of a productivity shock affecting marginal costs. This can be translated to prices more or less than proportionally, and so it affects entry (also this is in contrast to what happens with CES preferences usually assumed in macroeconomic theory). Therefore, supply shocks change the market structure generating additional processes of business creation/destruction.

erates pricing-to-market behavior and shows that trade integration shifts the production of some goods from the richer to the poorer country when the demand elasticity is increasing. Then, we move to the case of costly trade between identical countries, which affects the nature of the gains from variety compared to the Krugman model with CES preferences.

The work is organized as follows. Section 1 presents our model of monopolistic competition and characterizes its equilibrium. In Section 2 we consider the case for inefficient entry and discuss some extensions of the model. We extend the model to an outside good, heterogenous consumers and heterogenous firms in Section 3. In Section 4 we apply the model to study international trade. We conclude in Section 5.

1 The Model

Consider L identical agents consuming a mass of n goods under the following, symmetric, *indirect* utility function:

$$V(\mathbf{p}, E) = \int_0^n v\left(\frac{p_j}{E}\right) dj, \tag{1}$$

where $E > 0$ is the income of each agent to be spent in the differentiated goods and $\mathbf{p} > 0$ is the price vector.[4] The expression on the RHS of (1) exploits the homogeneity of degree zero of the indirect utility, and crucially assumes additive separability (i.e., "indirect additivity"). To satisfy sufficient conditions for (1) being an indirect utility function while allowing for a possibly finite choke-off price $\overline{s}$, we assume that the indirect sub-utility $v(s)$ is at least thrice differentiable, with $v''(s) > 0 > v'(s)$ for $s < \overline{s}$, and that $\lim_{s\to\overline{s}} v(s), v'(s) = 0$, with $v(s) = 0$ for $s \geq \overline{s}$. These assumptions imply that demand and extra utility are zero for a good that is not produced, i.e., that has a high enough price.

The Roy identity delivers the following (Marshallian) direct demand function of each consumer for good i:

$$x_i(\mathbf{p}, E) = \frac{v'\left(\frac{p_i}{E}\right)}{\int_j v'\left(\frac{p_j}{E}\right)\frac{p_j}{E}dj}, \tag{2}$$

which generates the total market demand $q_i = x_i(\mathbf{p}, E)L$. Notice that in the RHS of (2) the expression at the denominator, defined as $\mu(\mathbf{p}, E) < 0$, is the negative of the marginal utility of income, *times* the income level E.

Examples of (1) include simple cases such as the isoelastic function $v(s) = s^{1-\theta}$ with $\theta > 1$, the case of "mixtures" as $v(s) = s^{1-\alpha} + s^{1-\theta}$ with $\alpha > 1$, $\alpha \neq \theta$, the negative exponential function case with $v(s) = e^{-\tau s}$ and $\tau > 0$, and the "addilog" function $v(s) = (a-s)^{1+\gamma}$ with $a, \gamma > 0$.[5] Note that only if $v(\cdot)$ is

[4] As well known, any monotonic increasing and differentiable transformation of (1) is also a valid indirect utility. Using the wage as the *numeraire*, E can also be interpreted as the labor endowment of each agent (in efficiency units).

[5] Here the choke-off price $\overline{s} = a$ can be made arbitrary large.

isoelastic preferences are homothetic. Indeed, in such a case they are of the CES type, with indirect utility $V(\mathbf{p},E) = E\left(\int_j p_j^{1-\theta} dj\right)^{1/(1-\theta)}$ and θ equal to the elasticity of substitution. By an important duality result (see Hicks, 1969 and Samuelson, 1969), the case of CES preferences is the only one in which the class of preferences (1) satisfies direct addivitity as well. That is, it is the only case in which preferences can also be represented by an additively separable *direct* utility function, $U(\mathbf{x}) = \int_j u(x_j)\, dj$, as assumed in the D-S model. Therefore, the indirect utility (1) encompasses a class of (non-homothetic) preferences whose corresponding direct utility functions are non-additive.

To compare the assumptions of direct and indirect additivity, notice that indirect additivity amounts to assume that the consumption *ratio* of any two goods i and j, $x_i(\mathbf{p},E)/x_j(\mathbf{p},E)$, does not depend on the price of any other good, a rather intuitive concept. Under the D-S approach, on the contrary, it is the marginal rate of substitution between any two goods, $MRS_{i,j}(\mathbf{x}) = (\partial U(\mathbf{x})/\partial x_i)/(\partial U(\mathbf{x})/\partial x_j)$, which is independent of the consumption of other goods, leading to the property that their "inverse" price ratio, $p_i(\mathbf{x})/p_j(\mathbf{x})$, is independent from the quantities of the other goods. If both these properties are assumed to hold, then symmetric preferences must be of the CES type (see e.g. Blackorby *et al.*, 1978, Section 4.5.3).

Suppose now that each variety is sold by a (symmetric) firm producing with constant marginal cost $c > 0$ and fixed cost $F > 0$.[6] Following D-S, we model monopolistic competition by assuming that there are so many varieties that the impact of each individual price on the marginal utility of income is negligible.[7] Accordingly, the direct demand function for firm i in monopolistic competition is given by $q_i = v'(p_i/E)\, L/\mu$, where μ is taken as given, and profits can be written as:

$$\pi(p_i,E) = \frac{(p_i - c)v'\left(\frac{p_i}{E}\right) L}{\mu} - F. \tag{3}$$

The most relevant implication of this functional form is that the demand elasticity corresponds to the (absolute value of the) elasticity of $v'(\cdot)$, which we define as $\theta(s) \equiv -v''(s)s/v'(s) > 0$. This depends on the price as a fraction of income, p_i/E, but is independent of μ and L. Instead, in the primal case analyzed by D-S, the elasticity of *inverse* demand is uniquely determined by the consumption level.[8] This difference will be crucial for the analysis of the monopolistic competition equilibrium because market adjustments (needed to restore the zero-profit condition of endogenous entry) take place through shifts of demands due to changes in the number of firms which affect the marginal utility of income.

[6] As in Krugman (1980), where labor is used to produce goods and firms, by normalizing the wage to unity we can think of c and F as expressed in terms of labor units.

[7] Formally, the elasticity of μ with respect to p is of order $1/n$ if prices are not disproportionate: see the Appendix.

[8] In the D-S model the (individual) *inverse* demand of variety i is given by $p_i(x_i) = u(x_i)/\lambda$, where λ is the marginal utility of income (up to a monotonic transformation). Its elasticity is provided by $\sigma(x_i) = -u'(x_i)/(u''(x_i)x_i)$.

1.1 Equilibrium under monopolistic competition

Firm i maximizes (3) with respect to p_i. Its profit-maximizing price must then satisfy the FOC:

$$v'\left(\frac{p_i}{E}\right) + \frac{(p_i - c)v''\left(\frac{p_i}{E}\right)}{E} = 0, \tag{4}$$

with SOC $2v'' + (p_i - c)v'''/E > 0$. To satisfy (4) we assume that (locally) $v''s + v' > 0$, which implies $\theta > 1$ (this is equivalent to assume that goods are gross substitutes). If demand is (locally) concave ($v''' > 0$) the SOC is satisfied and $\theta' > 0$, that is, demand is perceived as more elastic when the price goes up. On the contrary, if demand is convex ($v''' < 0$) we may have $\theta' < 0$, that is, demand is perceived as less elastic when the price increases.

The FOC (4) can be rewritten as follows for the profit maximizing price p^e:

$$\frac{p^e - c}{p^e} = \frac{1}{\theta\left(\frac{p^e}{E}\right)}, \tag{5}$$

where the value of the Lerner index, the RHS of (5), is given by the reciprocal of the demand elasticity. To guarantee the existence of a solution to (5) we assume that $\overline{s}E > c$ (so that consumer willingness to pay is large enough) and that $\lim_{s\to\overline{s}} \theta(s) > \overline{s}E/(\overline{s}E - c)$.

The price rule (5) says that under indirect additivity the profit maximizing price is always independent from the number of varieties supplied, because this does not affect the demand elasticity. Notice that in the D-S model the profit-maximizing price depends, on the contrary, on the consumption level through an index of "relative love for variety" (Zhelobodko *et al.*, 2012), and therefore on the number of firms.[9] The comparative statics results with respect to E and c, however, fully depend on the sign of θ'. For instance, consider the case of $\theta' > 0$, which is arguably the most reasonable because it implies that the demand becomes more rigid when income increases. Then, the optimal price grows with income because firms face a more rigid demand (perhaps an entirely natural result). Similarly, it is easy to verify that a change in the marginal cost c is transmitted (pass-through) to prices in a less than proportional way (undershifting) if $\theta' > 0$. We summarize the previous results as follows:

PROPOSITION 1. *Under indirect additivity, monopolistic competition prices are independent from the number of firms and the size of the market, and increase in the income of consumers if and only if the demand elasticity is increasing with respect to the price.*

Since by symmetry the equilibrium profit is the same for all firms, and it is decreasing in their number, we can characterize the endogenous market structure through the following zero profit condition $(p - c)EL/np = F$. This and the pricing rule (5) deliver the free-entry number of firms and the production of

[9] The price rule of the D-S model is $(p - c)/p = 1/\sigma(x)$, where $1/\sigma(x)$ is the "relative love for variety" and $x = E/pn$ by the budget constraint (therefore n indirectly affects the price).

each firm:

$$n^{e}=\frac{EL}{F\theta\left(\frac{p^{e}}{E}\right)},\qquad q^{e}=F\frac{\theta\left(\frac{p^{e}}{E}\right)-1}{c}. \tag{6}$$

Notice that the equlibrium number of firms n^e is proportional to L/F, while q^e is proportional to F. These are well-known results of monopolistic competition with CES preferences that generalize to the whole class of preferences described by (1). In terms of elasticities, indirect additivity implies:

$$\frac{d\ln p}{d\ln L}=\frac{d\ln q}{d\ln L}=0 \quad \text{and} \quad \frac{d\ln n}{d\ln L}=1,$$

which is in striking contrast to what emerges in the general D-S model, where the comparative statics results of changes in L depend on the type of preferences.[10]

We conclude the comparative statics analysis of our market structure deriving the impact of changes in individual income and marginal cost:

$$\frac{d\ln p}{d\ln E}\gtreqless 0,\ \frac{d\ln q}{d\ln E}\lesseqgtr 0,\ \text{and}\ \frac{d\ln n}{d\ln E}\gtreqless 1 \ \text{ iff } \ \theta'(p^{e}/E)\gtreqless 0,$$

$$\frac{d\ln p}{d\ln c}\lesseqgtr 1,\ \frac{d\ln q}{d\ln c}\gtreqless -1 \ \text{and}\ \frac{d\ln n}{d\ln c}\lesseqgtr 0 \ \text{iff} \ \theta'(p^{e}/E)\gtreqless 0.$$

Accordingly, increasing individual income has an ambiguous effect on prices and firm size, depending on its impact on demand elasticity. The intuition is that when higher income makes demand more rigid ($\theta' > 0$), firms increase their prices and restrict production, which promotes business creation and increases the number of firms more than proportionally. The model generates the classic rationale for pricing-to-market: the same good should be sold at a higher price in richer markets where demand is more rigid, a result that is widely supported by wide empirical evidence in the trade literature (Alexandria and Kaboski, 2011; Fieler, 2011). The only work we are aware of, that investigates international pricing of identical goods and controls separately for income effects and market size effects is Simonovska (2010), who finds a elasticity of prices to country per capita GDP between 5% and 11% and does not find a significant impact of population on prices: this is in line with our theoretical findings.

Last, notice that since a change in c may affect prices more or less than proportionally, the marginal cost has an ambiguous impact on the number of firms which depends on the pattern of demand elasticity. For example, when

[10] The free-entry equilibrium conditions of the D-S model can be summarized as follows:

$$\frac{p^{e}-c}{p^{e}}=\frac{1}{\sigma\left(x^{e}\right)},\quad n^{e}=\frac{EL}{F\sigma\left(x^{e}\right)}\quad \text{and}\quad q^{e}=F\frac{\sigma\left(x^{e}\right)-1}{c}.$$

The impact of L depends on the sign of σ' (see Zhelobodko *et al.*, 2012). On the contrary, E is neutral on price and firm size, and increases the number of varieties proportionally. The reason of these different results is rooted in the market adjustment process. Since the profit expression in the primal approach is $\pi=\left(u'\left(x\right)/\lambda-c\right)Lx-F$, where $\lambda=\int_{j}u'\left(x_{j}\right)x_{j}dj/E$, there is a unique (symmetric) equilibrium (zero-profit) value of $\lambda=\left(nu'(x)x\right)/E$. On the contrary, under indirect additivity, there is a unique equilibrium value of $L/\mu=LE/\left[nv'\left(p/E\right)p\right]$.

demand elasticity is increasing ($\theta' > 0$), lower marginal costs are translated less than proportionally to prices, which increases the markups and attracts entry of new firms. Accordingly, and contrary to what happens with CES preferences, our general model suggests that demand shocks (i.e., affecting expenditure) and supply shocks (i.e., affecting marginal cost) generate additional processes of business creation/destruction. This should alter the dynamics of macroeconomic models with endogenous entry and monopolistic competition (Bilbiie *et al.*, 2012) or oligopolistic competition (Etro and Colciago, 2010). These results are summarized in the following propositions. The first one focuses on the number of firms:

PROPOSITION 2. *Under indirect additivity, in a monopolistic competition equilibrium with endogenous entry the number of firms increases proportionally with the size of the market; it decreases with respect to the marginal cost and increases more than proportionally with the income of consumers if and only if the demand elasticity is increasing with respect to the price.*

The next proposition summarizes the comparative statics on the production of each firm:

PROPOSITION 3. *Under indirect additivity, in a monopolistic competition equilibrium with endogenous entry the equilibrium production of each firm is independent from the size of the market, decreases with the income of consumers and decreases less than proportionally with respect to the marginal cost if and only if the demand elasticity is increasing with respect to the price.*

In conclusion, it is important to remark that our framework can be easily extended to the case of a finite number of firms and analyzed under Bertrand competition. In this case firms take strategic interactions into account and a traditional competition effect emerges. Then, a larger market size attracts more firms, which intensifies competition and reduces the markups: as a consequences the production of each firm increases and the number of firms increases less than proportionally with the market size.[11] This would match the evidence emphasized in the recent industrial organization and trade literature (see for instance Manuszak, 2002, Campbell and Hopenhayn, 2005, and Etro, 2014).

1.2 Examples and the relation with the direct utility

Our results can be illustrated in the following examples. The first is based on the (negative) exponential function $v(s) = e^{-\tau s}$, which generates the demand:

$$q_i = \frac{\tau e^{-\tau \frac{p_i}{E}}}{(-\mu)} L$$

[11] The Bertrand model generates the price:

$$\frac{p^B - c}{p^B} = \frac{1 + \left[\theta\left(p^B/E\right) - 1\right] F/EL}{\theta\left(\frac{p^B}{E}\right)}$$

which is decreasing in L. This model belongs to the class of aggregative games analyzed also by Acemoglu and Jensen (2011) with fixed number of firms and Anderson *et al.* (2012) with endogenous entry.

The free-entry equilibrium implies:[12]

$$p^e = c + \frac{E}{\tau}, \quad n^e = \frac{E^2 L}{F(c\tau + E)}, \quad q^e = \frac{F\tau}{E}. \tag{7}$$

The second example refers to the addilog case $v(s) = (a - s)^{1+\gamma}$, which delivers a linear demand when $\gamma = 1$:

$$q_i = \frac{2\left(a - \frac{p_i}{E}\right)}{(-\mu)} L$$

We obtain the following equilibrium values:

$$p^e = \frac{\gamma c}{1+\gamma} + \frac{aE}{1+\gamma}, \quad n^e = \frac{(aE - c)\, EL}{F\,(aE + \gamma c)}, \quad q^e = \frac{F(1+\gamma)}{aE - c}. \tag{8}$$

Notice that in both these examples $\theta' > 0$: therefore, growth in income makes demand more rigid, which leads firms to increase their prices and reduce their production, with a more than proportional increase in the number of firms. In addition, a marginal cost reduction is not fully translated to the prices, which attracts more business creation and has a limited impact on firm size: notice that q^e is unchanged in the exponential case and decreases in the addilog case.

At this point, one may wonder what kind of direct utility functions are behind our assumption of indirectly additive preferences. This can be done by using the Roy identity to obtain the inverse demand $p_i(\mathbf{x}) = Ev'^{-1}(\mu x_i)$ for each variety i. Employing the budget constraint $\int_j p_j(\mathbf{x}) x_j dj = E$, we obtain that $\mu(\mathbf{x})$ is implicitly defined from $1 = \int_j v'^{-1}(\mu x_j)\, x_j dj$ (notice that $\partial\mu/\partial x_i < 0$ under our assumption that $\theta > 1$). Simple expressions for μ arise if $v'^{-1}(\cdot)$ is homogenous or logarithmic (up to a linear transformation). This is the case of our two examples, where a closed-form solution for the inverse demand is available. With the exponential demand we obtain:

$$p_i(\mathbf{x}) = \frac{E}{\tau}\left[\ln(-\mu(\mathbf{x}))^{-1} - \ln x_i\right], \quad \text{where } \mu(\mathbf{x}) = -e^{-\frac{\tau + \int_j x_j \ln x_j dj}{\int_j x_j dj}},$$

and in the addilog case we have:

$$p_i(\mathbf{x}) = E\left[a - x_i^{1/\gamma}\left(\frac{-\mu(\mathbf{x})}{1+\gamma}\right)^{1/\gamma}\right], \quad \text{where } \mu(\mathbf{x}) = -(1+\gamma)\left[\frac{a\int_j x_j dj - 1}{\int_j x_j^{\frac{1+\gamma}{\gamma}} dj}\right]^{\gamma}.$$

Notice that μ depends on two simple aggregators: it is decreasing in total consumption and increasing in an index of consumption dispersion.[13]

[12] It appears useful to remark that our exponential demand function differs from the well-known Logit demand function, since the latter is characterized by constant demand rather than constant expenditure as ours.

[13] Of course, our previous results could be re-derived by assuming that each firm i takes μ as given and chooses its production level x_i to maximize $\pi_i = (p_i(\mathbf{x}) - c)\, Lx_i - F$. Cournot competition can be analyzed as well using the inverse demand systems discussed here: see Acemoglu and Jensen (2011) and Anderson *et al.* (2012) on the properties of aggregative games.

We can finally recover the direct utility function by plugging the inverse demand in the indirect utility (1). In general, this provides:

$$U(\mathbf{x}) = \int_0^n v\left(v'^{-1}(\mu(\mathbf{x})\, x_j)\right) dj = \int_0^n \widetilde{u}(\mu(\mathbf{x})\, x_j) dj, \tag{9}$$

where the "subutility" for each good $\widetilde{u}(\cdot)$ is increasing in the consumption x_i both directly and indirectly (by decreasing $\mu(\mathbf{x})$). In spite of this "additive" form, as we know the direct utility is not separable except for the CES case. In our two examples we obtain the following direct utility functions:

$$U(\mathbf{x}) = \sum_{j=1}^n x_i \exp\left(-\frac{\tau + \int_j x_j \ln x_j dj}{\int_j x_j dj}\right), \; U(\mathbf{x}) = \frac{\left(a \int_j x_j dj - 1\right)^{1+\gamma}}{\left(\int_j x_j^{\frac{1+\gamma}{\gamma}} dj\right)^{\gamma}} \tag{10}$$

which depend positively on total consumption and negatively on an index of consumption dispersion.

2 Estensions

In this section we extend the model to answer three different questions. The first is whether our results are robust to the introduction of an outside good representing the rest of the economy, as in standard general equilibrium models. The second is what is the role of heterogeneity between consumers and income distribution in affecting the properties of the equilibrium. The third is how cost heterogeneity between firms à la Melitz (2003) affects the endogenous entry process and the distribution of prices.

2.1 Outside good and optimality

Our baseline one-sector model can be extended to a two-sector model in which one sector is characterized by a homogenous good and the other one by differentiated goods. Following Dixit and Stiglitz (1977), we microfound it with an indirect utility that has a Cobb-Douglas form:[14]

$$V(p_0, \mathbf{p}, E) = \left(\frac{E}{p_0}\right)^{\gamma} \left(\int_0^n v\left(\frac{p_j}{E}\right) dj\right)^{1-\gamma}, \tag{11}$$

where $\mathbf{p}$ is the price vector of the differentiated goods, p_0 is the price of the outside good and $\gamma \in [0, 1)$. Clearly the model corresponds to the baseline one for $\gamma = 0$. In the Appendix we show that the pricing rule remains the same as in (5) and we prove the following proposition confirming our earlier results:

Proposition 4. *Ina two-sector economy with indirect additivity and monopolistic competition with endogenous entry in the differentiated sector, an*

[14] The case of a quasi-linear utility can be analyzed equivalently, but the current extension can be useful for general equilibrium applications.

increase in the size of the market is neutral on prices and increases linearly the number of firms, but higher income increases prices if and only if the demand elasticity is increasing with respect to the price.

In this more general set up it is interesting to evaluate the welfare properties of the equilibrium. As well known, firms do not fully internalize the impact of their entry decisions on the profits of competitors (a *business stealing* effect) and on the gains from variety (a *non-appropriability* effect), which may lead to too many or too few firms.[15] The constrained optimal allocation (maximizing utility under a zero profit constraint) crucially depends on the elasticity of the indirect sub-utility, $\eta(p/E) \equiv -v'(p/E)p/Ev(p/E) > 0$. In the Appendix we fully characterize optimality and prove the following result, that applies to both the baseline model and the two-sector one:

PROPOSITION 5. *Under indirect additivity, monopolistic competition with endogenous entry generates excess (insufficient) entry with too little (much) production for each firm if the elasticity of the indirect sub-utility $\eta(\cdot)$ is everywhere increasing (decreasing) with respect to the price.*

Paralleling D-S (p. 303), an intuition for this result can be obtained by noticing that η is approximately the ratio between the revenue of each firm and the additional utility generated by its variety. If $\eta' > (<)\ 0$ they diverge and at the margin each firm finds it more profitable to price higher (lower), i.e, to produce less (more), than what would be socially desirable. This, in turn, attracts too many (too few) firms.[16] In the Appendix we also characterize the first best allocation, which requires marginal cost pricing and subsidies to the firms.

2.2 Heterogeneous consumers

In this section we extend the baseline model to the case of consumers that have different incomes and different preferences. In spite of this realistic extension, the model remains tractable and allows to draw implications on the impact of income distribution on the market structure. We assume that the indirect utility of consumer $h = 1, 2, ..., L$ with income E_h is:

$$V_h(\mathbf{p}, E_h) = \int_0^n v_h\left(\frac{p_j}{E_h}\right) dj \tag{12}$$

In equilibrium each firm adopts a pricing rule depending on a weighted average of the demand elasticities of all consumers, where the weights are given by their fraction of total income. Changes in market size have the same impact as before, but now the distribution of income affects prices. In the Appendix we prove the following proposition:

[15] See Mankiw and Whinston (1986), Kuhn and Vives (1999) and Dhingra and Morrow (2012) for key references.

[16] One may find it more reasonable the case in which the elasticity of the subutility decreases with income, which requires $\eta' > 0$. Remarkably, this is the case for the negative exponential and addilog cases: accordingly, they both imply excess entry.

PROPOSITION 6. *Under indirect additivity with heterogenous consumers and monopolistic competition with endogenous entry, an increase in the size of the market is neutral on prices and increases linearly the number of firms; with identical preferences a change in the distribution of income in the sense of the first-order stochastic sense increases prices if and only if the demand elasticity is increasing with respect to the price.*

Suppose that all agents have the same preferences with increasing demand elasticity, but differ in income. In the Appendix we show that the equilibrium price is:

$$\frac{p^e - c}{p^e} = \frac{1}{\sum_{h=1}^{L} \omega_h \theta\left(\frac{p^e}{E_h}\right)} \tag{13}$$

where $\omega_h \equiv E_h/\Sigma_h E_h$ is the fraction of income of agent h. Then, consider a change in the distribution of income in the sense of the first-order stochastic sense. The new distribution increases the weight of high income agents in the average demand elasticity, reducing its value. This is going to increase the equilibrium prices, which in turn increases the number of firms and reduces the production of each one of them.

2.3 Heterogenous firms *à la* Melitz

The impact of changes in the size of the market or in the income of consumers should be more complex in the more realistic case of firms that are heterogenous in their technologies. Melitz (2003) has shown that under CES preferences and heterogenous costs there are no selection effects on the set of active firms when the size of the market increases, for instance due to costless trade with an identical country. However, this neutrality disappears under more general D-S preferences, that give raise to selection or anti-selection effects induced by an increase of the market size (see Zhelobodko *et al.*, 2012, and Bertoletti and Epifani, 2012). The purpose of this section is to study the effect of firms heterogeneity in our model of monopolistic competition based on indirect additivity, and to investigate whether selection effects emerge.

We assume that, upon paying a fixed entry cost F_e, each firm draws its marginal cost $c \in [\underline{c}, \infty)$ from a continuous cumulative distribution $G(c)$ with density $g(c)$ and $\underline{c} > 0$. In the Appendix we show that the price rule $p(c)$ is the same function of the marginal cost as before, and high-productivity firms produce more and are more profitable, but they also charge lower (higher) markups if θ is increasing (decreasing). Firms are active if they expect positive profits, that is if they have a marginal cost below the cut-off $\overline{c}$ satisfying:

$$[p(\overline{c}) - \overline{c}]\, v'(p(\overline{c})/E)L = \mu F \tag{14}$$

Finally, the mass of firms satisfies the endogenous entry condition:

$$\int_{\underline{c}}^{\overline{c}} [\pi(c) - F]\, dG(c) = F_e \tag{15}$$

This system determines $\bar{c}$ and μ in function of L, F, F_e and E, but in the Appendix we show that a change in the market size produces no selection effects. An increase of the population is completely neutral on prices and on the cut-off productivity level above which firms are active, even when preferences are not CES as in Melitz (2003), and it simply attracts a larger mass of firms.

Instead, besides increasing (decreasing) the mark up of the infra-marginal firms, a rise of income creates an anti-selection (selection) effect if θ increases (decreases) with respect to the price. In practice, in the natural case in which demand becomes more rigid with higher income, the consequence of an increase in per-capita income is that all firms increase their markups and prices, which attracts new and less efficient firms in the market. More formally we prove the following result:

PROPOSITION 7. *Under indirect additivity, monopolistic competition with endogenous entry and cost heterogeneity between firms, an increase in the size of the market is neutral on prices and on the set of active firms, but higher income increases all prices and makes less productive firms able to survive (an anti-selection effect) if and only if the demand elasticity is increasing with respect to the price.*

The main lesson is that the main results of the basic Melitz model extend to the more general microfoundation based on indirectly additive preferences, but the latter can provide new insights on the impact of income growth that are elusive with homothetic preferences.

3 International trade

One of the main limits of the modern trade models based on monopolistic competition *à la* Krugman (1980) and Melitz (2003) is their inability in generating reasons why firms tend to adopt different markups for their products in different countries. Nevertheless it is well known that pricing to market is a pervasive phenomenon in trade: in particular, identical products tend to have higher prices (net of the transport costs) in richer countries (Alessandria and Kaboski, 2011; Simonovska, 2010). In this section we apply our baseline model (with homogenous firms) to international trade issues, generalizing the celebrated Krugman (1980) model with CES preferences.[17]

We consider trade between two countries sharing the same preferences, given by (1), and technology, as embedded into the costs c and F, which are given in labor units, but possibly with different number of consumers and different income (namely productivity since the wage will be endogenous). Indexing the variables related to the "Foreign country" with a $*$, while keeping the same notation as above for the "Home country", we assume that the labor endowments of consumers in the Home and Foreign countries are respectively e and e^*, with $E = we$ and $E^* = w^*e^*$. Accordingly, the *nominal* marginal and fixed

[17] For recent analysis of trade with monopolistic competition and non-homothetic preferences see Fajgelbaum *et al.* (2011), Simonovska (2010) and Fieler (2011).

costs in the domestic and foreign countries are respectively wc and wF and w^*c and w^*F. Let us assume that to export each firm has to pay a symmetric "iceberg" cost $d \geq 1$. Consider the monopolistic competition profit of a firm i ($i = 1, ..., n$), based in the Home country, which has to choose its prices p_i and p_i^*:

$$\pi_i = \frac{(p_i - wc)v'\left(\frac{p_i}{E}\right)L}{\mu} + \frac{(p_i^* - dwc)v'\left(\frac{p_i^*}{E^*}\right)L^*}{\mu^*} - wF, \tag{16}$$

The corresponding profit for a firm i^* ($i^* = 1, ..., n^*$) based in the Foreign country that has to choose its prices p_{i^*} and $p_{i^*}^*$ is given by:

$$\pi_{i^*} = \frac{(p_{i^*} - dw^*c)v'\left(\frac{p_{i^*}}{E}\right)L}{\mu} + \frac{(p_{i^*}^* - w^*c)v'\left(\frac{p_{i^*}^*}{E^*}\right)L^*}{\mu^*} - w^*F, \tag{17}$$

where the marginal utilities of income in each country μ and μ^* are taken as given by each firm.

To characterize the symmetric equilibrium, let us denote variables related to exports with the index z. The pricing rules for the Home firms are:

$$\frac{p - wc}{p} = \frac{1}{\theta\left(\frac{p}{E}\right)}, \quad \frac{p_z - dwc}{p_z} = \frac{1}{\theta\left(\frac{p_z}{E^*}\right)} \tag{18}$$

and the pricing rules for the Foreign firms are:

$$\frac{p_z^* - dw^*c}{p_z^*} = \frac{1}{\theta\left(\frac{p_z^*}{E}\right)}, \quad \frac{p^* - w^*c}{p^*} = \frac{1}{\theta\left(\frac{p^*}{E^*}\right)} \tag{19}$$

so that four different prices are emerging in general, with $\mu = nv'\left(\frac{p}{E}\right)\frac{p}{E} + n^*v'\left(\frac{p_z^*}{E}\right)\frac{p_z^*}{E}$ and $\mu^* = nv'\left(\frac{p_z}{E^*}\right)\frac{p_z}{E^*} + n^*v'\left(\frac{p^*}{E^*}\right)\frac{p^*}{E^*}$ characterizing the marginal utility of income for the Home and Foreign consumers. The endogenous entry condition for the firms of the Home country reads as:

$$\frac{(p - wc)v'\left(\frac{p}{E}\right)L}{\mu} + \frac{(p_z - dwc)v'\left(\frac{p_z}{E^*}\right)L^*}{\mu^*} = wF,$$

and a corresponding one holds for the firms of the Foreign country. By using the pricing rules we can rewrite these conditions as:

$$\frac{pxL}{\theta\left(\frac{p}{E}\right)} + \frac{p_z x_z L^*}{\theta\left(\frac{p_z}{E^*}\right)} = wF \quad \text{and} \quad \frac{p_z^* x_z^* L}{\theta\left(\frac{p_z^*}{E}\right)} + \frac{p^* x^* L^*}{\theta\left(\frac{p^*}{E^*}\right)} = w^*F \tag{20}$$

To close the model we use the resource constraints (or, equivalently, the labor market clearing conditions):

$$eL = n\left[cxL + dcx_z L^* + F\right] \quad \text{and} \quad e^*L^* = n^*\left[dcx_z^* L + cx^*L^* + F\right], \tag{21}$$

and we can normalize the Home wage to unity. The general equilibrium system is hard to solve because equilibrium wages differ in the two countries. In particular, a country can have higher wages either because is larger or because it is more productive. However, we can easily characterize two polar cases that deliver the main insights of the model: the case without trade costs but with differences in both population and income between the two countries, and the one with costly trade between identical countries.

Let us first consider the case of costless trade between different countries, setting $d = 1$ and (possibly) $L \neq L^*$ and $e \neq e^*$. In the Appendix we prove the following result:

PROPOSITION 8. *Under indirect additivity, monopolistic competition with endogenous entry and costless trade between two countries different in population and income, the price of each good is higher when sold in the richer country and trade integration shifts the production of some goods from the richer to the poorer country if and only if the demand elasticity is increasing, without affecting the mark up.*

Since optimal pricing depends on local income, this case immediately generates pricing to market: prices of identical goods are higher in the richer country compared to the poorer one, independently from their market size, which is in line with the evidence presented by Simonovska (2010). Notice that this form of pricing to market cannot emerge within the D-S model under free entry. Agents benefit from consuming new goods produced abroad at the same price as the domestic goods, which creates pure gains from variety exactly as under CES preferences. Finally, costless integration induces a redistribution of firms and production across countries in spite of the fact that it does not affect the internal price levels: in particular, if $\theta' > 0$ the richer economy experiments a reduction in the number of firms and an increase in the firm size, which is intuitively due to the fact that some of the expensive internal sales are replaced by cheaper and larger sales abroad.

Consider now the opposite case of costly trade between identical countries, setting $d > 1$ but $L = L^*$ and $e = e^*$. In this case we prove:

PROPOSITION 9. *Under indirect additivity, monopolistic competition with endogenous entry and costly trade between two identical countries, domestic and export prices are the same in both countries, and trade integration reduces the number of firms if and only if the demand elasticity is increasing; trade opening decreases the average mark up if and only if the demand elasticity is increasing, and the (iceberg) trade costs have a non-monotonic impact on it.*

Since countries are identical, export prices are now the same in both countries. However, the model does not replicate the pure gains from variety that characterize the Krugman model with CES preferences, because the markup on export prices is not constant, but it is lower (higher) than the markup on domestic sales depending on whether $\theta' > (<)\ 0$. This affects the entry process as well: in equilibrium, the number of firms in each country decreases (increases) with respect to autarky if $\theta' > (<)\ 0$. Accordingly, in the realistic case of increasing elasticity, opening up to trade induces smaller gains from variety compared to the Krugman model but associated them with gains from reduced markups of the imported goods.

4 Conclusions

We have studied monopolistic competition under indirect additivity, a dual assumption with respect to the standard setting of Dixit and Stiglitz (1977). Our setting encompasses a number of new analitycally tractable cases as those with exponential or linear direct demands. The main properties of our market equilibrium coincide with those emerging under CES preferences for the impact of market size, but are entirely novel for the impact of changes in income, which allows for the emergency of pricing to market. We believe that further applications of the generalized model could be useful in macroeconomic analysis as well.

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Appendix

PROOF OF PROPOSITION 4. By Roy's identity the demand of each differentiated good becomes:

$$x_i = \frac{v'(p_i/E)}{\int_j \left[v'(p_j/E)\frac{p_j}{E} - \frac{\gamma}{1-\gamma} v(p_j/E) \right] dj}$$

and the expenditure in the outside good, $p_0 x_0$, does not depend on p_0.[18] Profits of each firm i are:

$$\pi_i = \frac{v'(p_i/E)(p_i - c)L}{\int_j \left[v'(p_j/E)\, p_j/E - \frac{\gamma}{1-\gamma} v(p_j/E) \right] dj} - F \tag{22}$$

[18] The intuitive reason, of course, is that preferences (11) are somewhat producing a unit intersector elasticity of substitution.

where the denominator μ is taken as given. It is immediate to verify that each firm adopts the same pricing rule as in (5), independent from the size of the market, γ and on the number of goods produced in the sector. The comparative static properties of the price with respect to E depend on the sign of θ' as before. Only the comparative statics of the equilibrium number of firms becomes more complex. Let us define $\eta(p/E) \equiv -v'(p/E)p/Ev(p/E) > 0$ as the absolute value of the elasticity of the indirect sub-utility $v(\cdot)$. Then, the number of goods produced in the free-entry equilibrium can be derived as:

$$n^e = \frac{EL\left(1-\gamma\right)\eta\left(p^e/E\right)}{F\left[\left(1-\gamma\right)\eta(p^e/E)+\gamma\right]\theta\left(p^e/E\right)} \tag{23}$$

which now depends on both the elasticities θ and η. Changes in income affect prices and the allocation of expenditure between the differentiated goods and the outside one. Nevertheless, the role of market size L remains the same as in the baseline model: in particular, doubling the market size does not affect prices but doubles the number of goods. □

Proof of Proposition 5. To compare the market performance with an optimal allocation we consider the constrained optimum, which maximizes utility under a zero profit condition for the firms. The problem boils down to:

$$\max_{n,p} nv\left(p/E\right) \ \ s.t. \ p = c + \frac{F}{x\left(p,n,E\right)L}.$$

The FOCs are:

$$\begin{aligned} v\left(p/E\right) &= -\rho\frac{F}{Lx^2}\frac{\partial x}{\partial n}, \\ nv'\left(p/E\right)/E &= -\rho\left[1+\frac{F}{Lx^2}\frac{\partial x}{\partial p}\right], \end{aligned}$$

which provides

$$\eta(p/E) = -\frac{\frac{xpL}{F}+\frac{\partial \ln x(p,n,E)}{\partial \ln p}}{\frac{\partial \ln x(p,n,E)}{\partial \ln n}} = \frac{p}{p-c}+\frac{\partial \ln x\left(p,n,E\right)}{\partial \ln p},$$

where ρ is the relevant Lagrange multiplier. Since

$$\frac{\partial \ln x\left(p,n,E\right)}{\partial \ln p} = \frac{\gamma\frac{d\ln\eta(p/E)}{d\ln p/E}}{\gamma+\left(1-\gamma\right)\eta\left(p/E\right)}-1$$

and $d\ln\eta/d\ln\left(p/E\right) = 1-\theta\left(p/E\right)+\eta\left(p/E\right)$, we obtain:

$$\frac{\partial \ln x\left(p,n,E\right)}{\partial \ln p} = \frac{\left(2\gamma-1\right)\eta\left(p/E\right)-\gamma\theta\left(p/E\right)}{\gamma+\left(1-\gamma\right)\eta\left(p/E\right)},$$

and accordingly:

$$\frac{p^* - c}{p^*} = \frac{\gamma + (1-\gamma)\,\eta\,(p^*/E)}{(1-\gamma)\,\eta\,(p^*/E)\,[1+\eta\,(p^*/E)] + \gamma\theta\,(p^*/E)}. \tag{24}$$

Finally the optimal number of firms is:

$$n^* = \frac{(1-\gamma)\,\eta LE}{F\,[(1-\gamma)\,\eta\,(1+\eta) + \gamma\theta]} \tag{25}$$

Under CES preferences $\partial \ln x / \partial \ln p = -1$ and the free-entry allocation corresponds to the constrained optimum. In the general case, the RHS of (24) is larger (smaller) than $1/\theta$ if (everywhere) $\theta\,(p/E) \gtrless 1 + \eta\,(p/E)$. It is also easily computed that:

$$\eta'(s) = \frac{\eta(s)\,[1 - \theta(s) + \eta(s)]}{s}, \tag{26}$$

Since it follows from (26) that $\eta' \lesseqgtr 0$ is equivalent to $1/\,(1 + \eta(p/E)) \gtrless 1/\theta(p/E)$, then (everywhere) $\eta' \lesseqgtr 0$ is equivalent to $n^e \lesseqgtr n^*$, which completes the proof. □

Remark. The unconstrained optimum assumes the availability of lump-sum transfers T to be used to subsidize each firm by an amount equal to LT/n. Such a first-best allocation solves the following problem:

$$\max_{n,p,T} \left(\frac{E-T}{p_0}\right)^{\gamma} \left(nv\left(\frac{p}{E-T}\right)\right)^{(1-\gamma)} \quad s.t. \;\; LT = n\,[F - (p-c)\,xL]\,,$$

where

$$x\,(p,n,E-T) = \frac{(E-T)\,(1-\gamma)\eta\,(p/E-T)}{np\,[(1-\gamma)\eta\,(p/E-T) + \gamma]}.$$

The solution requires marginal cost pricing, with $p^u = c$ and the following number of firms and transfer:

$$n^u = \frac{LE\,(1-\gamma)}{F\left[(1-\gamma)\,\eta\left(\frac{p^u}{E-T}\right) + 1\right]}, \tag{27}$$

$$T^u = \frac{E\,(1-\gamma)}{(1-\gamma)\,\eta\left(\frac{p^u}{E-T^u}\right) + 1}. \tag{28}$$

In general, relative to this first-best allocation, the market equilibrium with endogenous entry may have too few or too many firms: notice, however, that $n^u > n^* = n^e$ if preferences are CES. □

Proof of Proposition 6. The demand of a consumer h for good i can be now derived as:

$$x_{hi}(\mathbf{p}, E_h) = \frac{v_h'\left(\frac{p_i}{E_h}\right) E_h}{\int_j v_h'\left(\frac{p_j}{E_h}\right) p_j dj}.$$

We exclude any form of price discrimination by the firms, so each firm has to choose a single price for all the consumers. Therefore, the profits of firm i can be written as:

$$\pi(p_i) = (p_i - c)\sum_{h=1}^{L}\frac{v_h'\left(\frac{p_i}{E_h}\right)}{\mu_h} - F,$$

where $\mu_h = \int_j v_h'\left(\frac{p_j}{E_h}\right)\frac{p_j}{E_h}dj$ and therefore also the marginal utility of income of each agent are taken as given. Let us define θ_h as the demand elasticity of consumer h, $\bar{E} = \Sigma_h E_h$ as the aggregate income of the agents and $\omega_h \equiv E_h/\bar{E}$ as the fraction of income of agent h. Deriving the optimality condition and imposing symmetric pricing we obtain:

$$\frac{p^e - c}{p^e} = \frac{1}{\sum_{h=1}^{L}\omega_h\theta_h\left(\frac{p^e}{E_h}\right)}$$

The Lerner index must equate the inverse of a weighted average of the demand elasticities of all consumers, where the weights are given by their fraction of total income. Endogenous entry implies the following number of firms:

$$n^e = \frac{\bar{E}}{F\sum_{h=1}^{L}\omega_h\theta_h\left(\frac{p^e}{E_h}\right)}.$$

Notice that the distribution of income is not neutral with respect to prices and number of firms when consumers are heterogenous. For example, suppose that all agents have the same preferences with increasing demand elasticity, but differ in income. Then, consider a change in the distribution of income in the sense of the first-order stochastic sense. The new distribution increases the weight of high income agents in the average demand elasticity, reducing its value. This is going to increase the equilibrium prices, which in turn increases the number of firms and reduces the production of each one of them. □

Proof of Proposition 7. Let us start analyzing the price choices for the active firms: it is immediate to verify that the same pricing rule (5) applies to each of them. Let us denote with $p = p(c)$ the profit-maximizing price of a c-firm, with $x(c) = v'(p/E)/\mu$ the individual consumption of its product, and with:

$$\pi(c) = [p(c) - c]\, v'(p/E)L/\mu$$

its variable profit for given μ. Note that the optimal price of firm c does not depend upon L and μ. The first-order and second-order conditions for profit maximization imply the following elasticities with respect to the marginal cost:

$$\frac{\partial \ln p(c)}{\partial \ln c} = \frac{\theta\left(p\left(c\right)/E\right) - 1}{2\theta\left(p\left(c\right)/E\right) - \zeta\left(p\left(c\right)/E\right)} \gtreqless 1 \Leftrightarrow \theta' \gtreqless 0, \tag{29}$$

$$\frac{\partial \ln x(c)}{\partial \ln c} = -\theta\left(p\left(c\right)/E\right)\frac{\partial \ln p(c)}{\partial \ln c} < 0, \tag{30}$$

$$\frac{\partial \ln p(c)x(c)}{\partial \ln c} = \frac{-\left(\theta\left(p\left(c\right)/E\right)-1\right)^2}{2\theta\left(p\left(c\right)/E\right)-\zeta\left(p\left(c\right)/E\right)} < 0, \tag{31}$$

$$\frac{\partial \ln \pi(c)}{\partial \ln c} = 1-\theta\left(p\left(c\right)/E\right) < 0. \tag{32}$$

Accordingly, high-productivity (low-c) firms are larger, make more revenues, and are more profitable, as in Melitz (2003). Unlike the Melitz model, however, now larger firms charge lower (higher) markups[19] if θ is increasing (decreasing), as can be seen from (29).

In addition, always taking as given μ, we have the following elasticities with respect to income:

$$\frac{\partial \ln p(c)}{\partial \ln E} = \frac{\theta\left(p\left(c\right)/E\right)+1-\zeta\left(p\left(c\right)/E\right)}{2\theta\left(p\left(c\right)/E\right)-\zeta\left(p\left(c\right)/E\right)} \gtreqless 0 \Leftrightarrow \theta' \gtreqless 0, \tag{33}$$

$$\frac{\partial \ln x(c)}{\partial \ln E} = \frac{\theta\left(p\left(c\right)/E\right)^2-\theta\left(p\left(c\right)/E\right)}{2\theta\left(p\left(c\right)/E\right)-\zeta\left(p\left(c\right)/E\right)} > 0, \tag{34}$$

$$\frac{\partial \ln p(c)x(c)}{\partial \ln E} = \frac{\theta\left(p\left(c\right)/E\right)^2+1-\zeta\left(p\left(c\right)/E\right)}{2\theta\left(p\left(c\right)/E\right)-\zeta\left(p\left(c\right)/E\right)} > 1, \tag{35}$$

$$\frac{\partial \ln \pi(c)}{\partial \ln E} = \theta\left(p\left(c\right)/E\right) > 1. \tag{36}$$

Notice that the size of each firm increases with E (for given μ), and revenues and profits increase more than proportionally. However, each price increases with respect to income only when $\theta' > 0$, and decreases otherwise.

The set of active firms is the set of firms productive enough to obtain positive profits from their profit-maximizing pricing strategy. Denote by $\overline{c}$ the marginal cost cutoff, namely the value of c satisfying the zero cutoff profit condition $\pi(\overline{c}) = F$, or:

$$\left[p(\overline{c})-\overline{c}\right]v'(p(\overline{c})/E)L = \mu F. \tag{37}$$

The relation (37) implicitly defines $\overline{c} = \overline{c}(E, \mu F/L)$. Differentiating it yields:

$$\frac{\partial \ln \overline{c}}{\partial \ln E} = \frac{\theta\left(p\left(\overline{c}\right)/E\right)}{\theta\left(p\left(\overline{c}\right)/E\right)-1} > 0, \tag{38}$$

$$\frac{\partial \overline{c}}{\partial \mu}\frac{\mu}{\overline{c}} = \frac{\partial \ln \overline{c}}{\partial \ln F} = -\frac{\partial \ln \overline{c}}{\partial \ln L} = \frac{1}{1-\theta\left(p\left(\overline{c}\right)/E\right)} < 0. \tag{39}$$

Endogenous entry of firms in the market implies that expected profits

$$\pi^E = \int_{\underline{c}}^{\overline{c}} \left[\pi(c)-F\right]dG(c) \tag{40}$$

[19] We define the mark up of a c-firm as $\theta\left(p\left(c\right)/E\right)/\left(\theta\left(p\left(c\right)/E\right)-1\right)$.

must be equal to the sunk entry cost F_e. The profits decrease when the absolute value of μ increases, that is $\partial\pi^E/\partial\mu > 0$. Accordingly, the condition $\pi^E = F_e$ pins down uniquely the equilibrium value of μ as a function $\mu(E, L, F, F_e)$. In particular, using (37) the free entry condition can be written as:

$$\int_{\underline{c}}^{\overline{c}} \left\{ \frac{[p(c) - c]\, v'(p(c)/E)}{[p(\overline{c}) - \overline{c}]\, v'(p(\overline{c})/E)} - 1 \right\} dG(c) = \frac{F_e}{F} \tag{41}$$

The system (37)-(41) can actually be seen as determining $\overline{c}$ and μ in function of L, F, F_e and E. The second equation determines $\overline{c}$ independently from the market size L and the first one determines μ as linear with respect to L. The cut-off $\overline{c}$ is therefore independent of market size, because μ proportionally adjusts in such a way to keep constant the ratio L/μ and thus the variable profit of each firm. As a consequence, as in Melitz (2003), a change in the market size L produces no selection effect, even when preferences are not CES. Also notice that a raise of F requires an increase of μ less than proportional (otherwise the value of the expected variable profit would increase more than proportionally), and this in turn decreases $\overline{c}$ (a selection effect), while an increase of F_e by increasing the equilibrium value of μ raises $\overline{c}$ (an anti-selection effect).

The impact of income E is more complex. Since by (36) and (38) an increase of E raises π^E, it must decrease the equilibrium value of μ. In particular:

$$\frac{\partial\mu}{\partial E}\frac{E}{\mu} = -\frac{\frac{\partial\pi^E}{\partial E}}{\frac{\partial\pi^E}{\partial\mu}}\frac{E}{\mu} = \overline{\theta}\left(\overline{c}\right) > 0, \tag{42}$$

where

$$\overline{\theta}\left(\overline{c}\right) = \left[\int_{\underline{c}}^{\overline{c}} \frac{1}{\theta\left(p\left(c\right)/E\right)} \frac{p(c)x(c)}{\int_{\underline{c}}^{\overline{c}} p(c)x(c)dG(c)} dG(c) \right]^{-1} \tag{43}$$

is the harmonic mean of the θ values according to $G(\cdot)$ and $\overline{c}$. Computing the total derivative of $\overline{c}$ with respect to E we obtain:

$$\begin{aligned} \frac{d\ln\overline{c}}{d\ln E} &= \left[\frac{\partial\overline{c}}{\partial E} + \frac{\partial\overline{c}}{\partial\mu}\frac{\partial\mu}{\partial E}\right]\frac{E}{\overline{c}} = \frac{\theta\left(p\left(\overline{c}\right)/E\right)}{\theta\left(p\left(\overline{c}\right)/E\right) - 1} + \frac{\overline{\theta}\left(\overline{c}\right)}{1 - \theta\left(p\left(\overline{c}\right)/E\right)} \\ &= \frac{\theta\left(p\left(\overline{c}\right)/E\right) - \overline{\theta}\left(\overline{c}\right)}{\theta\left(p\left(\overline{c}\right)/E\right) - 1}. \end{aligned} \tag{44}$$

It follows that $d\overline{c}/dE \gtrless 0$ if (everywhere) $\theta' \gtrless 0$: that is, in addition of increasing (decreasing) the mark up of the infra-marginal firms, a rise of E creates an anti-selection (selection) effect if θ increases (decreases) with respect to the price.

To close the model, the measure of active firms n is determined by the budget constraint, requiring average expenditure to equal E/n, and thus:

$$\frac{E}{n} = \int_{\underline{c}}^{\overline{c}} p(c)x(c)\frac{dG(c)}{G(\overline{c})}. \tag{45}$$

Since an increase of the market size L affects proportionally μ, and thus proportionally reduces individual consumption $x(c)$, it follows from (45) that it also proportionally increases the number of varieties. By (35) and (**??**) one can obtain:

$$\begin{aligned}\frac{d\ln\{p(c)x(c)\}}{d\ln E} &= \frac{\partial\ln\{p(c)x(c)\}}{\partial\ln E}+\left[\frac{\partial x(c)}{\partial\mu}\frac{\mu}{x(c)}\right]\left[\frac{\partial\mu}{\partial E}\frac{E}{\mu}\right]\\ &= \frac{\theta\left(p\left(c\right)/E\right)^2+1-\zeta\left(p\left(c\right)/E\right)}{2\theta\left(p\left(c\right)/E\right)-\zeta\left(p\left(c\right)/E\right)}-\overline{\theta}\left(\overline{c}\right) \qquad (46)\end{aligned}$$

where $\zeta(s)\equiv -v'''(s)s/v''(s)$ is a measure of curvature of perceived demand. Notice that an equivalent (local) condition for the SOC to be satisfied is $2\theta>\zeta$. Since $t(\theta)=\left(\theta^2+1-\zeta\right)/\left(2\theta-\zeta\right)$ is such that $t(\theta)\leq\theta$ and $t(1-\zeta)=\theta$, (46) is null in the CES case, implying that a change of E affects proportionally the number of firms. Outside the CES case we cannot determine unambiguously the impact of E on the number of goods.[20] □

Proof of Proposition 8. Let us assume $d=1$ with $L\neq L^*$ and $e\neq e^*$. In such a case each firm faces the same demand functions, independently from the country in which it is based. However, the firms based in the Home country have a cost advantage (disadvantage) with respect to firms from the Foreign country if $w<(>)\ w^*$. Since a necessary condition for a monopolistic equilibrium with endogenous entry in both countries is $\pi=\pi^*=0$, it follows that it must be $w/w^*=1$. Accordingly, we can normalize the common wage to $w=w^*=1$, which restores the notation of the previous sections with $E=e$ and $E^*=e^*$, and conclude that in equilibrium $p=p_z^*$ and $p^*=p_z$. This means that all firms adopt the same price in the same country, with:

$$\frac{p-c}{p}=\frac{1}{\theta\left(\frac{p}{E}\right)},\qquad \frac{p^*-c}{p^*}=\frac{1}{\theta\left(\frac{p^*}{E^*}\right)}, \qquad (48)$$

[20] Notice, however, that by rewriting (45) as $EG(\overline{c})=n\int_{\underline{c}}^{\overline{c}}p(c)x(c)dG(c)$, and totally differetiating, we have:

$$\frac{d\ln n}{d\ln E}G(\overline{c})=G(\overline{c})+Eg(\overline{c})\frac{d\overline{c}}{dE}-np(\overline{c})x(\overline{c})g(\overline{c})\frac{d\overline{c}}{dE}-n\int_{\underline{c}}^{\overline{c}}\left[\frac{d\ln\{p(c)x(c)\}}{d\ln E}\frac{p(c)x(c)}{E}\right]dG(c)$$

from which:

$$\begin{aligned}\frac{d\ln n}{d\ln E} &= 1+\frac{g(\overline{c})}{G(\overline{c})}\frac{\theta\left(p\left(\overline{c};E\right)/E\right)-\overline{\theta}\left(\overline{c};E\right)}{\left(\theta\left(p\left(\overline{c};E\right)/E\right)-1\right)}\frac{\overline{c}}{E}\left[E-np(\overline{c};E)x(\overline{c};E)\right]\\ &\quad+\left\{\overline{\theta}\left(\overline{c};E\right)-\int_{\underline{c}}^{\overline{c}}t\left(\theta\left(p(c;E)/E\right)\right)\frac{p(c;E)x(c;E)}{E/n}\frac{dG(c)}{G(\overline{c})}\right\}. \qquad (47)\end{aligned}$$

The square brackets on the RHS of the last expression is positive by (**??**): accordingly, the second term on the RHS is positive (and the elasticity of n with respect to E is, ceteris paribus, larger) if and only if $\theta'>0$. However, the last term is just the difference between the harmonic mean of θ and the average of $t(\theta)$, and thus its value ought to depend on the distribution of θ equilibrium values, which in turn depends on $v(\cdot)$ and $G(\cdot)$.

where $p > p^*$ if $E > E^*$ and $\theta' > 0$. Notice that $\mu/\mu^* = v'(p/E)\, pE^*/v'(p^*/E^*)\, p^*E$, and accordingly $\mu > \mu^*$ if and only if $E > E^*$. On the other side, the opening of costless trade has in this case no impact on the mark up (no competitive effect), which remains the same as in autarky, extending this property of the Krugman (1980) model to our entire class of underlying preferences.

From symmetry we infer that all firms have the same profit:

$$\pi = \frac{EL/\theta\,(p/E) + E^*L^*/\theta\,(p^*/E^*)}{n+n^*} - F : \tag{49}$$

and that the zero-profit constraint pins down the total number of firms as:

$$n + n^* = \frac{EL}{F\theta\left(\frac{p}{E}\right)} + \frac{E^*L^*}{F\theta\left(\frac{p^*}{E^*}\right)}, \tag{50}$$

This is the same as the total number of firms emerging under autarky in each country. Consumers benefit from consuming new goods produced abroad at the same price as the domestic goods, which creates pure gains from variety. Finally, by using the resource constraints one can obtain

$$\frac{n}{n^*} = \frac{EL}{E^*L^*}. \tag{51}$$

Thus, if $E \neq E^*$ and $\theta' \neq 0$ costless integration induces a redistribution of firms and production across countries in spite of the fact that it does not affect the internal price levels. In particular, if $\theta' > 0$ the richer economy experiments a reduction in the number of firms and an increase in the firm size, which is intuitively due to the fact that some of the expensive internal sales are replaced by cheaper and larger sales abroad. □

Proof of Proposition 9. Let us assume $d > 1$ but $L = L^*$ and $e = e^*$. In such a case, all the equilibrium variables must be the same across countries by symmetry. Therefore we can again normalize $w = w^* = 1$, which implies that $E = E^*$. The internal prices and the prices of exports must be the same in both countries, i.e., $p = p^*$ and $p_z = p_z^*$. These prices satisfy:

$$\frac{p-c}{p} = \frac{1}{\theta\left(\frac{p}{E}\right)}, \qquad \frac{p_z - dc}{p_z} = \frac{1}{\theta\left(\frac{p_z}{E}\right)}.$$

Moreover, by using $E = n\,(px + p_z x_z)$ and (20), the number of firms in each country can be derived as follows:

$$n = \frac{EL}{F}\left[\theta\left(\frac{p}{E}\right)^{-1}\frac{px}{px+p_zx_z} + \theta\left(\frac{p_z}{E}\right)^{-1}\frac{p_zx_z}{px+p_zx_z}\right]. \tag{52}$$

Notice that the parenthesis in (52) is a weighted average of $1/\theta\,(p/E)$ and $1/\theta\,(p_z/E)$. Therefore, the number of firms in each country decreases (increases) with respect to autarky if $\theta' > (<)\ 0$. This outcome differs from the traditional one in the Krugman (1980) model with trade costs, where CES preferences imply that opening up to trade with an identical country always doubles the number of consumed varieties. However, the price of the imported goods is higher, with a mark up that on average decreases (increases) if $\theta' > (<) 0$. Finally, in this setting the impact of d on the "average" mark up and on the number of firms must be non-monotonic (an effect noted for the primal setting by Bertoletti and Epifani, 2012). In fact, when d is so large to make negligible the exports (and their expenditure share), the price and the number of firms go back to their autarky levels, which coincide with those of costless integration (as we have seen above). $\square$