Optimal asset allocation aid system: from “one-size” vs “tailor-made” performance ratio

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November 22nd, 2006

Abstract

Optimal asset allocation well-fitting the investors goals is a pressing challenge in risk management. In spite of Sharpe ratio, Sortino-Satchell, Generalized Rachev and Farinelli-Tibiletti ratios are new parameter-dependent performance ratios able to suit the investor risk profile. Five investor prototypes are studied and fifteen tailor-made optimal asset paths are traced over a rolling twelve month investing horizon. For almost all investor prototypes, simulations proved superiority of personalized ratios respect to Sharpe ratio in guiding to higher performance. Ex-post controlling on compatibility of optimal asset strategies with ex-ante investor risk profiles have done satisfactorily results.

JEL Subject Classification: C8, G0, G1, G8

Key words: Risk management; Decision support system; Asset allocation.
1 Introduction

How to choose the best decision support system for making optimal asset allocation well-fitting investors goals is an evergreen challenge in Risk Management. In 1966 it was developed a ratio (see Sharpe, 1966), originally named reward-to-variability, giving the trade-off between the expected return and standard deviation. A plain procedure was suggested: the greater the ratio, the better the performance. Subsequently, drawbacks of Sharpe ratio for non-gaussian assets had been highlighted and new performance indexes for asymmetrical distributions were introduced (see Biglova et al. (2005), Sortino and Satchell (2001)). To capture downside-risk, standard deviation was replaced by VaR, CVaR and partial moments of different orders and numerous ex-post empirical investigations questioning what the best performer ratio was, have been carried out (see Biglova et al. (2004b)).

Recently a new challenge is pressing practitioners. Strategical asset allocation is not only expected to drive to the best cumulated wealth, but also be tailored to the investor risk profile. So, performance ratio should not only take advantage on the peculiar assets distribution features (skewness and leptokurtosis etc.) but also to meet personalized goals. Clearly, a first approach to hit this point is imposing sector constrains to each risky asset categories.

Our task is to go a step forward using also personalized decision aid systems. The dependent performance ratios seem to service our request. Specifically we focused on Sortino-Satchell, Generalized Rachev ratios (see Biglova et al.(2004a,b) and Farinelli-Tibiletti (see Farinelli and Tibiletti (2003b, 2006) and Menn et al. (2005) pages 208-209). By a proper parameter balancing, ratios can be crafted to investor profile interpreting its subjective attitude toward the upside and downside deviations from the benchmark. We considered five investor prototypes and fifteen parameter-dependent ratios plus the Sharpe ratio taken as a comparison ratio.

For sensitivity analysis purposes we assumed an active investment strategy for optimal asset allocation, for each ratio under consideration. Since joint distributions of prospective returns are unknown, empirical analysis have been carried out. Historical monthly data covering the period January 29, 1993 to October, 31, 2005 of eight stock indexes show evidence of high volatility in returns. To empirically forecast expected returns and covariance matrix Exponential Weighted Moving Average approach (EWMA) is applied.

1 The views expressed herein are those of the authors and do not necessarily reflect opinions shared by UBS and Cantonal Bank of Zurich.
For each investor prototype, our task was twofold:

- comparing the return paths obtained through three different parameter-dependent ratios and the Sharpe ratio used as reference point.
- controlling whether optimal allocation over the investing period be conformed with ex-ante investor risk profile.

Simulations confirmed our conjecture that ratios based on personalized one-sided risk measures lead to most satisfactorily final wealth respect to the Sharpe ratio, for all investor prototype under consideration. Moreover, if parameter-dependent ratios are used, empirical results confirmed the matching between the optimal allocations with the ex-ante risk-profile. Whereas as the Sharpe ratio is used, optimal allocation is not conformed to a clear-cut risk-profile.

The paper is organized as follows. In Section 2 different investor profiles are presented. How proper parameters can be crafted to the investor profile is discussed. Section 3 designs optimal asset allocation paths according to 15 parameter-dependent ratios. Backtests are carried over a 12 month rolling period. At the end, tests on compatibility of optimal allocations with ex-ante risk profiles are made. Conclusion and final remarks are collected in Section 4.

2 From “one-size” vs “tailor made” reward-risk performance ratios

As usual in practice, investors are classified into different risk profile prototypes, specifically we concern:

(1) *defensive*, if investor is seeking stability and she is less concerning about growth of final wealth;

(2) *conservative*, if she is seeking stability with modest potential for increased investment value;

(3) *moderate*, if she is a long-term investor and he is seeking steady growth potential without the need for current income;

(4) *growth*, if she is a long-term investor seeking good growth potential;

(5) *aggressive* if she is a long-term investor seeking high growth potential.
A common way to ensure well-fitting allocation is to impose sector constrains to asset categories in portfolio. As mentioned before, our task is to go a step forward in tailored investing, by using personalized performance ratios.

2.1 From Sharpe ratio vs parameter-dependent ratios

Let $R$ be a $p$-integrable random variable, i.e., $E[|R|^p] < \infty$, $p > 0$, in $L^p$ space labelling the total return over a fixed time horizon for a certain investment. A reward-risk performance index is nothing but the trade-off between the reward and risk. The most common ratio is the so-called Sharpe ratio, where the reward and risk are given by the mean and standard deviation of $R$, respectively. In 1994 the Sharpe ratio (Sharpe, 1994) was extended to be referred to a benchmark $b \in \mathbb{R}$, as:

$$
\Phi(R) = \frac{E(R - b)}{\text{st.dev.} (R - b)}.
$$

It is worthwhile noting that in this case upside and downside deviations to the benchmark are equally weighted. Clearly, if our concern is measuring the stability around the benchmark, this approach suits the case. that may be no longer true if our concern is having the trade-off between favorable/unfavorable deviations (see Farinelli and Tibiletti (1993)).

In addition to Sharpe ratio, we focus on different allocation paths derived by three classes of parameter-dependent one-sided ratios: (1) Sortino-Satchell, (2) Generalized Rachev and (3) Farinelli-Tibiletti ratios.

- **Sortino-Satchell ratio.** Instead of standard deviation, downside semi-variance is used. This is a measurement of return deviation below a minimal acceptable rate, called MAR. The ratio is only penalizing for “harmful” volatility. It has been subsequently generalized into the Sortino-Satchell Ratio (2001):

  $$
  \Phi_{SS}(R; q; b) := \frac{E(R - b)}{E^{1/q}[(R - b)^-]^q}
  $$

  where the risk measure is a left partial order moment of the excess return $R - b$ and MAR works as the benchmark $b$.

- **Generalized Rachev ratio.** Instead of measuring over- and underperformance with respect to the benchmark, Generalized Rachev ratios (see Biglova et al. (2004a,b))
pose the attention on the left and right tail of the excess portfolio return distribution, as follows

\[ \Phi_{GR}(R) := \frac{\mathbb{E}[(R^+)^\gamma | R \geq -VaR_{1-\alpha}(R)]}{\mathbb{E}[(R^-)^\delta | R \leq -VaR_{\beta}(R)]} \]

where \( \alpha, \beta \in (0, 1) \), the parameters \( \gamma, \delta > 0 \) and \( VaR_c(R) := -\inf \{ x | \mathbb{P}(R \leq x) > c \} \) is the Value-at-Risk of \( R \).

- **Farinelli-Tibiletti ratio.** To measure the trade-off between good/bad volatility, Farinelli-Tibiletti ratio uses partial moments of different orders (see Farinelli and Tibiletti (2003b, 2006) and Menn et al. (2005) pages 208-209)

\[ \Phi_{FT}(R; \theta; b) := \frac{\mathbb{E}^{1/p}[(R-b)^+]^p}{\mathbb{E}^{1/q}[(R-b)^-]^q} \]

where \( \theta := (p, q) \) with \( p, q > 0 \).

- **Omega Index.** If \( p=q=1 \), the index \( \Phi_{FT} \) reduces to

\[ \Omega_b(R) = \Phi_{FT}(R; 1; b) = \frac{\mathbb{E}[(R-b)^+]}{\mathbb{E}[(R-b)^-]} \]

an index introduced in Keating-Shadwick (2002) (see Farinelli and Tibiletti (2006)). Since both orders are equal to one, small and large deviations from the benchmark are equally weighted. Kazemi et al. (2004) show that Omega Index can be also interpreted as the ratio between the price of an European call option written on the investment and the price of an European put option written on the investment.

Balancing parameters \( p \) and \( q \) agent’s attitude toward consequences of over performing or under performing can shaped. It is known that the higher \( p \) and \( q \), the higher the agent’s preference for (in the case of expected gains) or dislike of (in the case of expected losses) the extreme events. This highlight can be even traced back to Fishburn (1977). If agent’s main concern is that she might fail the target without particular regard to the amount, then a small value (i.e. \( q < 1 \)) for the left order is appropriate. But, if small deviations below the benchmark are relatively harmless when compared to large deviations (catastrophic events), then a large value (i.e. \( q > 1 \)) for the left order is recommended. The choice of the proper right order \( p \) is made analogously and should capture the relative appreciation for outcomes above the benchmark. If moderate gains are desired with respect to an exceptional performance, a low right order (i.e. \( p < 1 \)) is indicated. On the other hand, a high \( p > 1 \) should describe the opposite attitude. Parameters \( \gamma \) and \( \delta \) in Generalized Ratio behave the same as \( p \) and \( q \) in Farinelli-Tibiletti ratio.
3 Portfolio optimization

An integrated decision system for asset allocation follows. At a first step, a statistical analysis of asset returns is carried out for possible non-gaussian distributed assets for which raw historical data are unable to estimate the reward and risk in the objective function \( \Phi(\cdot) \) to be optimized. After this procedure, the performance ratios have to be maximized. This reads in finite dimension as:

\[
\max_{w \in \mathcal{W}} \Phi_j(w'r)
\]  

(3.2)

where

\[
\mathcal{W} = \left\{ w \in \mathbb{R}^n \mid \sum w_i = 1, w_{\min} \leq w \leq w_{\max} \right\}
\]

Here \( \Phi(X) := \frac{r(X)}{\rho(X)} \) denotes the performance measure, \( X := w'r - b \) the excess return, \( b \) the benchmark and \( r \) the vector-valued random variable for the asset returns with historical values \( \{r_t\} \). Since we refer to a set of fifteen performance ratios, the subscript in \( \Phi(\cdot) \) varies as \( j = 1, \ldots, 15 \). Notice that \( t = -T, \ldots, 0, 1, \ldots, h \), where \( T \) denotes the size of the sample data and \( h \) is the length of the reference planning horizon for the portfolio manager. Each component \( r_{i,t} \) of vector \( r \), represents the random return of the asset \( i = 1, \ldots, N \) at time \( t \). The entire planning horizon is divided in \( K \) unit periods, each of length \( h/K \). We iterate the problem (3.2) \( K \) times and treat it as a sequence of one period stochastic optimization problems.\(^2\)

4 Numerical Backtests

4.1 Personalized parameters

A key point in personalized allocation is to be aware of parameter influence on the optimal strategy. Keeping that in mind we compared optimal allocation paths for above mentioned five risk profiles obtained through 15 parameter-dependent ratios:

- **(1) defensive investor**: \( p = 0, 5 \) and \( q = 2 \) (and equivalently \( \gamma = 0, 5 \) and \( \delta = 2 \));
- **(2) conservative investor**: \( p = 1, 5 \) and \( q = 2 \) (and equivalently \( \gamma = 1, 5 \) and \( \delta = 2 \));
- **(3) moderate investor**: \( p = q = 1 \) (and equivalently \( \gamma = \delta = 1 \));

\(^2\)This is a good approximation of the dynamic structure of the optimization problem. Indeed we use historical simulation together with an econometric forecasting model to solve the uncertainty in the objective function.
(4) *growth investor*: $p = 2$ and $q = 1, 5$ (and equivalently $\gamma = 2$ and $\delta = 1, 5$);

(5) *aggressive investor*: $p = 3$ and $q = 0, 5$ (and equivalently $\gamma = 2$ and $\delta = 0, 5$).

For defensive and conservative we used the same Sortino-Satchell with $q = 2$.

### 4.2 Investment categories

Four investment categories are taken under consideration: (1) Bonds; (2) Hedge Funds; (3) Property Equities; (4) Equities. Going from the lower to the higher level of risk. For each risk category two indexes are selected, so $N = 8$ total return indexes are involved:

<table>
<thead>
<tr>
<th>$i$</th>
<th>Asset</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>JPM US government bond</td>
<td>Bond</td>
</tr>
<tr>
<td>2</td>
<td>JPM EUROPE government bond</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>HENNESSEE International</td>
<td>Hedge Fund</td>
</tr>
<tr>
<td>4</td>
<td>HENNESSEE Macro</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>FTSE EPRA/NAREIT Europe</td>
<td>Property Equity</td>
</tr>
<tr>
<td>6</td>
<td>NAREIT Equity</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>MSCI North America</td>
<td>Equity</td>
</tr>
<tr>
<td>8</td>
<td>MSCI Europe</td>
<td></td>
</tr>
</tbody>
</table>

The benchmark used as a proxy of the risk free rate be the 3 month US INTERBANK offered rate ($i = 9$).

Historical data of total returns and the benchmark covering from January 29, 1993 to October 31, 2005, for a total of 154 observations, are used to simulate the portfolio performance. No leverage is allowed, and operational restrictions to the percentages of wealth to invest are imposed: $\mathbf{w}_{\text{min}} = (0.02, 0.02, 0.02, 0.02, 0.02, 0.02, 0.02, 0.02)$, $\mathbf{w}_{\text{max}} = (0.3, 0.3, 0.3, 0.3, 0.3, 0.3, 0.3, 0.3)$. As we are focusing on parameters sensitivity analysis, we do not impose further constrains to investment categories.

Statistical analysis show evidence of non-gaussian data with weak stationarity.\(^3\) We decided to employ the Exponential Weighted Moving Average (EWMA) estimation procedure for the volatilities and correlations among the eight assets in the portfolio. As

\(^3\)The augmented Dickey-Fuller test of unit root for the univariate time series in $r_t$ indicates non-stationarity. The Ljung-Box test results in the acceptance of the null hypotheses of no serial correlations. The Jarque-Bera and the Mulnor tests for respectively the univariate and the multivariate time series confirm the data non-normality.
a forecasting model for the vector of expected returns and the covariance matrix, this model is a reasonable approximation of more sophisticated Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models especially when we deal with financial data with low frequency, such as the monthly series employed in this example. Although the EWMA is a constant volatility model, with weakly stationary and non-gaussian data it can be used in the optimization procedure.

Using the algorithm described in Section 3, the optimal paths of cumulated wealth over the 12 month horizon are calculated for Sharpe ratio and Sortino-Satchell, Generalized Rachev, Farinelli-Tibiletti ratios. Parameter settings used are referred to five investor prototypes (see Section 2). In conclusion, we compare fourteen performance ratios in all 4.

Our investigation is carried out for each investor prototype with a double objective:

- comparing ratios ability in guiding to the best profitable final wealth;
- checking whether optimal asset diversification during the investing period is well modulated to the ex-ante investor risk profile.

With reference to the former objective, our investigation involved: (1) defensive (2) conservative (3) moderate (4) growth (5) aggressive investors, where personalized parameters are specified in the boxes. Figures 1-5 contain optimal cumulative wealth paths.

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4We use 4 parameter configurations for the Sortino-Satchell ratio and 10 parameter settings for the Farinelli-Tibiletti ratio and the Generalized Rachev ratio.
Figures 1-5 show that all ratios raised final wealth. Sharpe ratio led always to underperformed allocations, whereas Sortino and Farinelli-Tibiletti ratios are almost always performance leaders for all investor profiles. Specifically, Sortino ratio with parameter $q = 0.5$ for an aggressive investor prototype has beaten the record in final wealth among all the investors categories. Empirical tests confirmed superiority of ratios based on one-sided risk measures to Sharpe ratio in setting profitable investments.

Nevertheless, it is worthwhile pinpointing an advice. If the optimal asset allocation induced by a tailor-made ratio does not lead to the best final cumulative return, the "culprit" of a possible underperformance is not imputable to a low forecasting ability of the ratio itself. But rather to the chosen ex-ante investor profile that may have refrained to take advantage of profitable market wages.
Latter investigation aims at controlling compatibility of ex-post optimal asset diversification with ex-ante risk profile chosen. Figures 6 to 20 collect optimal weights for the rolling-period exposure, \textit{i.e.}, the time series of weights for the eight assets involved.
Figure 6: Optimal portfolios with Sharpe ratio

Figure 7: Optimal portfolios for defensive and conservative profile: Sortino-Satchell

Figure 8: Optimal portfolios for defensive profile: Generalized-Rachev
Figure 9: Optimal portfolios for defensive profile: Farinelli-Tibiletti

Figure 10: Optimal portfolios for conservative profile: Generalized-Rachev

Figure 11: Optimal portfolios for conservative profile: Farinelli-Tibiletti
Figure 12: Optimal portfolios for moderate profile: Sortino-Satchell

Figure 13: Optimal portfolios for moderate profile: Generalized-Rachev

Figure 14: Optimal portfolios for moderate profile: Farinelli-Tibiletti
Figure 15: Optimal portfolios for growth profile: Sortino-Satchell

Figure 16: Optimal portfolios for growth profile: Generalized-Rachev

Figure 17: Optimal portfolios for growth profile: Farinelli-Tibiletti
Figure 18: Optimal portfolios for aggressive profile: Sortino-Satchell

Figure 19: Optimal portfolios for aggressive profile: Generalized-Rachev
Figure 20: Optimal portfolios for aggressive profile: Farinelli-Tibiletti

Weight path evolutions over time have shown how different risk profiles and parameter-dependent ratios may guide to different optimal allocations. In general, we get a confirmation of standard guide-lines in custom-tailored allocation: as investors move from a defensive profile to a more aggressive one, also optimal weight-mix moves from a low-risk asset overweighting (bonds, i.e. assets n.1-2), to higher-risk investments (equities, i.e. assets n.5-6 and hedge funds, i.e. assets n.7-8). Some remarks can be drawn:

- In months 2-3-4 Sharpe ratio and parameter-dependent ratios seem to produce similar investments leading to equally-balanced asset mix. Whereas, differences stick out in the remaining nine months, where equities (assets 7-8) and property equities (assets 5-6) have a relevant weight only in aggressive and growth portfolios.

- In accordance to the fact that Generalized Rachev-ratio is mostly focused on the extreme distribution tails, in the case of defensive profile (see Figure 8) it leads to the mostly bond-overweight portfolio, whereas in the case of aggressive profile a steady equity-overweight can be found.

- Sharpe ratio seems to mimic a defensive profile during months 1 and 7 guiding to a bond-overweight portfolio, which is similar to those of defensive parameter-dependent ratios. Nevertheless, at month 9 and 12 Sharpe ratio switches to an equity-overweight portfolio.

In conclusion, Sharpe ratio does not fit a clear-cut risk profile over the global investing period. Whereas, all parameter-dependent ratios well-suit all ex-ante chosen risk profiles. Generalized Rachev-ratio for a defensive investor (see Figure 8) displays the most steady bond-overweight composition for the all twelve-month period, whereas Sortino-Satchell
and Farinelli-Tibiletti ratios prefer to switch at final epoch from bonds (assets 1-2) to hedge funds (assets 3-4).

5 Conclusion

Customized optimal asset allocation is faced for five different investor prototypes. After having fixed sector constrains to each asset categories in portfolio, personalized parameters for the Sortino-Satchell, Generalized Rachev and Farinelli-Tibiletti ratios are chosen and optimal investing paths carried out for a twelve-month investing period. Superiority of parameter-dependent ratios in achieving higher performance with respect to Sharpe ratio is empirically proved. Specifically, Sortino-Satchell and Farinelli-Tibiletti ratios adapted to a moderate aggressive investor, outperformed the best. At the end, ex-post control on compatibility of investor risk profiles with optimal allocations is carried over. All parameter-dependent ratios successfully came up to expectations, whereas the Sharpe ratio seemed not to interpret a clear-cut and stable investor profile. Because of our results are strongly influenced by the asset universe and investing horizon, further simulations are needed.

References


