Bounded Rationality and Heterogeneity in Economic Dynamic Models

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Chapter 1: General introduction and thesis outline

Chapter 1

General introduction and thesis outline

Economic phenomena are characterized by a strong mutual dependence between the choices of economic actors, e.g. individual consumers or firms, and the economic environment in which these choices are made and evaluated. The origin of this dependence lies in the fact that the economic environment is formed by the aggregation of all individual choices, while at the same time individual choices are affected by the overall economic environment.

In order to clarify the nature of this dependence, consider the following examples which will be recurrent throughout this thesis. In a market of a perishable consumption good, firms have to decide today how much to produce to supply to the market tomorrow. On the one hand, firms base their production decision upon tomorrow's expected profits. On the other hand, tomorrow's profits will depend on the total amount firms produce. In fact, given consumer demand, profits are a function of market clearing prices, which depend on total supply. Financial markets are another well-known example of this mutual dependence between choices and environment. The demand of investors for an asset is driven by expected future returns, while at the same time asset returns are determined, through realized asset prices, by investors' demand. Another example, with the advantage of appealing to everyone's experience, is the *El Farol* problem proposed by Arthur (1994). *El Farol* is a bar where a live music show takes place on a fixed day, say Thursday, every week. There are 100 people who would like to go to the bar on Thursday evening, but the *El Farol* bar is not big enough for all of them. In fact, the bar is too crowded when there are more than 60 people. Each Thursday evening everyone decides whether to go to the bar or not. Each decision maker is directly influencing the number of agents present at the bar, and at the same time, since that number affects his or her utility, expectations about the number of agents present at the bar influence each individual decision. Notice that in all these examples the mutual dependence between individual decisions and economic environment is mediated through a feedback relation between expectations and realizations of some variable, e.g. prices, returns, or attendance level. For this reason we can call these systems *expectational feedback systems*.

Traditional analysis in economics, i.e. neoclassical economics, has circumvented the complications arising through the type of interaction of many individuals outlined above, by assuming that all agents are *rational*. Modeling human behavior as rational implies that, in making the decision that maximizes their objectives, agents take into account the choices of others, assuming that others are doing the same. When expectations are involved, agents are actually supposed to solve the expectational feedback system and find a "fixed point" or equilibrium outcome where expectations and realizations of the same variable coincide. Assuming rational thinking leads directly to an equilibrium where choices need not be corrected, unless unanticipated changes of the exogenous parameters characterizing the environment or the decision makers, take place. Thus, if agents were rational, observed changes of economic variables, which are a function of agents' choices, should come from a response to unexpected changes in some exogenous characteristics, or fundamentals, of the economy. Empirical analysis, both of real world data as well as of laboratory experiments with human subjects, is at odds with this statement and shows that economic variables fluctuate, even when changes in the fundamentals of the economy do not occur.

In this thesis we investigate the possibility that economic fluctuations can be explained through the *interaction of boundedly rational agents*, that is, agents are not assumed to be rational and are not necessarily able to solve the mutual dependence implied by the expectational feedback. Boundedly rational agents use simple behavioral rules and adapt their behavior over time, switching from time to time to better performing rules. Since it is not a priori clear which of these simpler rules should be used, we explicitly assume that agents are *heterogeneous* and employ different rules to address the same decision problem. We let a selection mechanism, such as "survival of the fittest", discipline the class of behavioral rules.

In departing from the traditional rational approach we have two main goals. First, we want to appraise if an argument used in favor of rationality, namely that rationality is the outcome of the repeated interaction of heterogeneous boundedly rational agents, is justified. This part of the analysis has thus a theoretical motivation. Second, having shown in what respect our results differ from the rational benchmark, we want to characterize whether our interacting agents framework can reproduce empirically observed phenomena in the specific economic settings we consider.

In the context of this general background, this thesis is built around three different models discussed in three independent chapters. Every model refers to one of the three economic frameworks briefly sketched at the beginning of this introduction, for which the expectational feedback is a crucial characteristic. Chapter 2 uses a classical cobweb model, i.e. a market for a perishable consumption good, to analyze the trade-off between benefits and costs of rules with different degrees of sophistication. In fact, if sophisticated predictions, as rational expectations, are costly, agents could decide to adopt a cheaper, but simpler predictor, as long as its predictions are not too far from the realized values. We investigate the impact on commodity prices of the interaction of agents having this extra degree of freedom in choosing between cheap free riding and costly sophisticated prediction. In Chapter 3, we characterize the competition of a large group of boundedly rational agents using different strategies to repeatedly exploit the same scarce resource, when no market institutions are present to coordinate agents' actions. This chapter has been inspired directly by Arthur's *El Farol* bar problem. We compare our results with those of equilibrium rational solutions, agent-based computational simulations, and laboratory experiment with human subjects. In Chapter 4, we investigate whether, in financial markets, the interaction of boundedly rational agents, triggered by informational differences, can help explaining time series properties of empirical financial data.

In the remaining part of this introductory chapter, Section 1.1 offers a more detailed review of the theoretical background of this thesis, i.e. bounded rationality and interaction of heterogeneous agents. Section 1.2 presents each of the three remaining chapters. In that section we also offer a brief introduction to nonlinear dynamics, the main technical tool that will be used throughout this thesis. Section 1.3 concludes this introductory chapter discussing an interesting common feature of our models, namely the presence of *negative* expectational feedback. This feature proves to be helpful to interpret chapters' results.

1.1 Theoretical background

Economic theory has always faced the issue of how to model human behavior, in particular how to model individual decision making in an interactive framework. Assuming that an economic actor, e.g. an individual consumer or a firm, wants to make the decision in his best interest and that he is capable of making this judgment, the two approaches that are currently in use rely on different levels of the decision maker's *rationality*.

The approach of neoclassical economics, which we will refer to as *rational*, can be summarized using two requirements. First, the decision maker is assumed to be able to choose the alternative that maximizes utility or profit, given his beliefs about the economic environment and the actions of the other actors in the economy. Second, each decision maker is assumed to be able to predict exogenous as well as endogenous variables, so that his original beliefs are self-fulfilling. The first requirement is mostly related to the rationality of preferences and choices. The second requirement is mostly related to the rationality of expectations, that is, of agents' prediction of future variables, including other agents' actions. An implication of these two requirements is that a rational decision maker knows as much as the modeler regarding the economic framework of interest. Moreover, the rational agent is also assumed to be able to "solve" this model, that is, to make decisions such that all predictions and beliefs are consistent with the outcome of all agents' choices. Stated differently a rational agent is supposed to solve for the equilibrium of the expectational feedback system. Thus, the primary effort of a rational agent consists of searching for an equilibrium[1]. At the equilibrium rational agents do not need to revise their decisions unless an exogenous change in some variables of the model, such as an unanticipated change in the agents' preferences or in the structure of the economy, comes about.

The *bounded rationality* approach (see e.g. Conlisk, 1996 for a survey) considers the requirements that rationality poses on peoples' characteristics, both in terms of knowledge of the economic environment and of their computational capabilities, unrealistic because they are too demanding. Different notions of bounded rationality have been formulated in the literature. For example, in Simon (1957) and in Rubinstein (1998) the emphasis is on limitations of human knowledge and computational abilities in decision making, whereas Sargent (1993) questions the second aspect of rationality, that is the capability of individuals to form rational expectations. Generally speaking, a boundedly rational agent is modeled as able to choose what he perceives as the best for himself, but he does not know the exact structure of the environment. Put differently, we can think of a boundedly rational decision maker as one choosing only from a set of alternatives which is bounded by his individual perception. In particular, when predictions of future variables or choices of other economic agents are involved, bounded rationality implies that ex-ante predictions and ex-post outcomes need not coincide perfectly. Therefore a boundedly rational agent is not assumed to be able to "solve" the model. That is, he is not able to choose equilibrium outcomes where all beliefs are self-fulfilling so that his decisions are, both ex-ante and ex-post, optimal. On the contrary, a boundedly rational agent uses simple rules of thumb and keeps revising his choices as he learns about the economic environment in which he is operating, through feedback about his past decisions. In particular, expectational feedbacks relates agents' expectations about relevant variables,

[1]When more equilibria are presents, it remains to be seen if agents are able to coordinate on the same equilibrium. Coordination games are the simplest example of this problem. We do not raise this important issue in this thesis as in all the model we investigate only one equilibrium typically stands out. See also Section 1.3 of this introdusction

e.g. prices, to their realizations. As a result of the feedback structure and learning, models of bounded rationality are structurally dynamic and may or may not settle down to an equilibrium where agents have learned how to coordinate their actions.

The decision whether to model agents as rational or boundedly rational is part of the assumptions of an economic model. The scholars opting for rationality argue that it is a helpful assumption to describe the equilibrium outcome of people's economic interaction. This defence of rationality is known as the *"as if"* argument and goes back to Friedman and Savage (1948), Friedman (1953), and Alchian (1950). In particular, Friedman (1953) argues that a model should not be judged in terms of the realism of its assumptions but in terms of the realism of its predictions, and that modeling agents as rational is justified because the repeated interaction of heterogeneous boundedly rational agents leads to the same outcome *as if* agents are rational. The general underlying idea is that agents who are not rational would learn to be rational over time since incentives to behave rationally, such as higher profits or utility, are constantly at work. In fact, agents who adopt non-rational rules would be out-performed by agents using rational rules, since rational rules come from optimization and lead to higher economic performance. In summary, assuming rationality is often based on the presumption that this approach offers the equilibrium outcome of repeated interaction, the details of which are not worth being modeled.

Part of the recent interest in models of bounded rationality has been motivated by the attempt to put some structure on the *"as if"* argument. Does the interaction of boundedly rational agents leads to the same outcome *as if* agents are rational? Convergence to rational behavior has been the object of investigation of many theoretical papers in the last two decades. In macroeconomics, Sargent (1993) and Evans and Honkapohja (2001) address the possibility of agents learning to form rational expectations. The main message is that "... some rational expectations equilibria are learnable while others are not. Furthermore, convergence will in general depend on all economic parameters of the system, including policy parameters" (Bullard 2006, p. 205). Learnability of equilibria thus needs to be investigated case by case. In microeconomics, more specifically in game theory, a related issue is the possibility of learning Nash equilibria, i.e. the equilibria played by rational agents, as investigated in Fudenberg and Levine (1998). Evolutionary game theory explores the *"as if"* argument when the process of converging is regulated by evolutionary forces driven by "survival of the fittest", rather than by adaptive learning, see e.g. Weibull (1995). Whereas, according to the *"as if"* argument agents who are closer to rationality should make larger profits and thus overcome other types of behavior, cases exist, as summarized e.g. in Weibull (1994), where convergence is not attained and fluctuations around the equilibrium never vanish. In this thesis we will also encounter some examples of this behavior.

Together from appraising whether modeling the interaction of different groups of boundedly rational agents supports the *"as if"* argument, modeling agents as boundedly rational can be an important step in narrowing the differences between the predictions of economic theory and empirical data. Much evidence has been collected in the last thirty years against the practice of modeling human behavior as rational. For example, Conlisk (1996) classifies this evidence as either direct, through rationality tests on individuals, or indirect, when models assuming rational agents are at odds with empirical data. Direct evidence against rationality consists, for example, in showing that, when faced with decisions involving uncertainty which have an objectively correct answer, agents show psychological biases and failure in rationalizing the problem (see e.g. Tversky and Kahneman, 1974). As already argued by Simon (1957), agents rather use simple rules of thumb or heuristics than engage in difficult, more rational, computations. Moreover, when deciding whether to use more sophisticated rules, agents take into account deliberation costs, such as information gathering costs or information processing costs as those costs associated with the extra effort they are putting into the decision making. Indirect evidence against rationality has been collected from empirical testing of economic models built under the assumption that agents are rational. Consider the following examples that are related to the economic frameworks investigated in the rest of this thesis. In experiments of market entry games, which are similar to the *El Farol* game discussed previously, at the aggregate level agents seem to be able to coordinate on a Nash equilibrium of the game, but, at the individual level, use different simple rules which do not coincide with rational behavior. Furthermore, their interaction generates excess variability of the entry percentages with respect to the game theoretical predictions, (see e.g. Sundali, Rapoport, and Seale, 1995 or Ochs, 1990). In financial markets, prices seem to be much more volatile than justified by the movement of the underlying fundamentals (see e.g. Shiller, 1989) and returns are correlated (see e.g. Fama and French, 1988b). Moreover, expectational surveys, as Frankel and Froot (1987) or Chow (1989), argue that agents do not use rational expectations. However in general, it is still under debate whether macroeconomic fluctuations of unemployment, business cycles and growth rates are partly driven by expectations as argued in Grandmont (1985), see also Grandmont (1998) and Hommes (2004).

In deviating from rationality, and modeling agents as boundedly rational it is often assumed that agents are *heterogeneous*. Kirman (1992, 2006) summarizes some of the reasons why the assumption that agents are homogeneous, or that their heterogeneity is not relevant as their choices can be summarized by the choice of a so-called representative agent, should be discarded. One commonly referred to reason is the "no trade" argument, which states that homogeneous agents, as agents with homogeneous expectations, would have no reasons to trade among themselves. Another reason, which is particularly relevant to our theoretical background, involves an evolutionary explanation. That is, evolution and adaptation necessarily requires some variety of behavior, if one wants the selection mechanism to indicate which is the "surviving" trait. Denying heterogeneity in models with boundedly rational agents gives rise to limitations as it is not clear a priori which kind of boundedly rational behavior each agents should be endowed with. For these reasons, modeling agents as heterogeneous is becoming more and more popular in economics, as shown e.g. by the extensive surveys on analytical and computational models with heterogeneous agents in Hommes (2006) and LeBaron (2006).

In this thesis we assume that agents are heterogeneous, in the sense that they choose different simple decision rules to address the same decision problem. Generally speaking, rules can differ in terms of sophistication, where the most sophisticated rule corresponds to rationality. We also assume that the higher the sophistication of a rule, the higher the deliberation cost an agent pays in order to use it. Rules can also differ in terms of the information they use, where information can also be costly. Instead of considering fixed fractions of agents employing each rule, we let them evolve over time as a function of their "fitness". Thus, we employ an evolutionary approach where a "survival of the fittest" mechanism is at work. A rule that has performed better according to some measure, to be defined case by case, is used by a higher fraction of agents. In our models, we use two different updating mechanism for fractions. In Chapter 2 we use a discrete choice mechanism along the lines of Manski and McFadden (1981) and Anderson, de Palma, and Thisse (1993) (see also Brock and Hommes, 1997, for an early application). This updating mechanism assumes that in choosing between different strategies, agents have an idiosyncratic component that, together with the fitness measure, determines how individual choices are distributed among the different alternatives. In Chapters 3 and 4, we use the replicator dynamics of Taylor and Jonker (1978). The replicator dynamics is related to biological reproduction and the number of agents using a certain rule evolves both as a function of the current number of agents using that rule, and of the fitness of each rule. The replicator dynamics can also be motivated in the context of boundedly rational agents learning and imitating strategies in a strategic environment (see e.g. Weibull, 1995, Chapter 5 or Binmore and Samuelson, 1997).

1.2 Thesis Outline

This thesis is built around three main economic frameworks, which are developed in separate chapters. Each chapter is self-contained, with its own introduction, conclusion, notes, and appendices as needed. For this reason each chapter can be read independently from the others. A common bibliography is collected at the end of the thesis. A working paper has been extracted from each chapter: Brock, Dindo, and Hommes (2006) is based on Chapter 2, Dindo and Tuinstra (2006) on Chapter 3 and Diks and Dindo (2006) on Chapter 4. This section briefly discusses each chapter and the mathematical tools used for their analysis.

1.2.1 Deliberation costs in a cobweb model

In Chapter 2, we use the classic example of the cobweb model (see e.g. Ezekiel, 1938) to investigate the impact of endogenizing agents' choices between a costly sophisticated prediction rule, such as rational expectations, and a cheap prediction rule, such as naive expectations.

The cobweb model describes the production decisions of a producer of perishable consumption goods which take one period to be produced, such as crop or cattle. Producers want to maximize their next period's profit, which depends on the next period's market price of the good. Assume that the production technology is convex so that optimal output is an increasing function of the agent's prediction of market price. Also assume, as usual, that the aggregate demand function is downward sloping. In this case market equilibrium implies that a high (low) supply leads to a low (high) market price. Summarizing, high (low) expected prices result in a high (low) supply which clears the demand at a low (high) realized price in this system. The characteristic of this system is that the ex-post realized price is "opposite" to the ex-ante expected price.

This example offers a typical case of an economic system with a mutual dependence between individual choices and aggregate outcome. The dependence is due to the fact that producers' expectations of prices affect realized prices. When the ex-ante expected price is equal to the average ex-post realized price, that is, when the expectational feedback is at a fixed point, we have an expectational equilibrium price.

Historically, the cobweb model is an important example, because Muth (1961) uses this framework to introduce the concept of rational expectations. Muth argues that rational agents, in order to effectively optimize realized profits, should use a rational expectations predictor. This is the same as the expectational equilibrium or the "prediction of the relevant economic theory" (Muth, 1961, p. 316). Muth shows that in a cobweb model, the rational expectations predictor is the one with the highest realized profits. In other words, rational expectations are optimal expectations. Agents using other predictors would perform worse than those using rational expectations, and would, sooner or later, be wiped out of the market. However, Muth assumes that agents can choose among all possible predictors, rational or not, at no cost. Muth ignores the presence of deliberation costs, or information gathering costs, associated with more sophisticated prediction rules.

In Chapter 2, we concentrate on the cobweb model taking into account deliberation costs. Early contributions along the same line are Conlisk (1980), Sethi and Franke (1995), and Brock and Hommes (1997). In particular, in their paper Brock and Hommes introduce the concept of *adaptive rational equilibrium dynamics (ARED)*, where agents choose between a costly rational expectations forecast and a cheap naive forecast. The fractions of agents using each of the two strategies evolve over time and are endogenously coupled to the market equilibrium price dynamics. Brock and Hommes show that when the selection pressure to switch to the more profitable strategy is high, instability and complicated chaotic price fluctuations arise. Brock and Hommes call this phenomenon a rational route to randomness.

In their setting, Brock and Hommes assume that agents are *backward looking* in the sense that strategy selection is based on *experience* measured by relative past realized profits. Implicitly this means that agents, even those employing rational expectations concerning prices, use naive expectations regarding the amount of profit earned by each of the strategies. In fact, in deciding which predictor is best at maximizing expected profit, they use today's profit as a forecast of expected profit.

In Chapter 2, we model the ARED with *forward looking* agents, that is, where strategy selection is based upon *expected* profits rather than *realized* profits. As agents' objective is to maximize expected profits, assuming that agents choose a strategy based upon expected profitability seems a natural extension of the original model by Brock and Hommes. Our aim is to investigate whether forward looking behavior dampens, fosters or eliminates price fluctuations compared to backward looking behavior. In particular we analyze whether forward looking behavior has an impact on the rational route to randomness found by Brock and Hommes.

A second contribution of Chapter 2 consists in establishing an equivalence between a heterogeneous agents model with switching between two different strategies, and a representative agent framework, where the representative agent optimally chooses from a continuum of alternative predictors. As usual, predictors differ for their cost and degree of sophistication. This analysis aims at finding a correspondence between the mechanism responsible for the updating of predictor choices in a heterogeneous agents framework, and the cost function associated with a continuum of predictors in a representative agent framework. Notice that if such a correspondence exists, price fluctuations driven by strategy switching of heterogeneous firms may as well being explained by a representative firm switching between a continuum of predictors with different characteristics.

1.2.2 Competition and coordination in participation games

Chapter 3 is devoted to an analysis of repeated interaction of a large number of boundedly rational agents that are competing for the same scarce resource, when no coordinating market institution is at work. We formalize this general interaction structure, which has been inspired by the *El Farol* game, as a participation game. We concentrate on participation games for which the payoff for participating decreases as the number of participating agents increases, so that there is negative expectational feedback. Well-known examples of participation games with negative feedback analyzed in the literature are market entry games, where firms have to decide whether to enter a market and compete, or stay out of it. Another example is given by route choice games, where a group of commuters repeatedly choose, between two routes, the fastest way from their homes to their offices.

Experimental research in this area (see e.g. Sundali, Rapoport, and Seale, 1995) has aimed at appraising to which extent the aggregate participation rate emerging from the competition of many agents can be described using the traditional tools of game theory. The evidence is mixed. At the aggregate level the time average participation rate is consistent with the symmetric Nash equilibrium. However, at the individual level agents do not learn to play that Nash equilibrium, but use different deterministic rules. In particular individuals seem to employ simple rules, such as always participate, always abstain, or participate conditionally on the outcome of previous rounds. As a result the aggregate participation rate is much more volatile than would be in the case where all agents play Nash. The computational model of Arthur (1994), where 100 heterogeneous agents are choosing among different decision rules to decide whether to participate or not, gives the same results. In fact, Arthur observes convergence of the first moment of the participation rate to the symmetric Nash equilibrium, but he obtains a higher second moment, and thus a more volatile participation rate series.

Our aim is to obtain a simple analytic model that can replicate the main experimental and computational findings in the area of participation games with negative feedback. At this purpose we use the same model building guidelines as Brock and Hommes (1997) and Droste, Hommes, and Tuinstra (2002). We set up an analytic model with heterogeneous boundedly rational agents choosing between simple rules. Fractions of agents using each rule are endogenous and evolve according to the past performance of each rule as described by the replicator dynamics. We concentrate on the evolutionary dynamics produced by the competition between different deterministic rules that condition the participation decision on the outcome of the previous rounds. We characterize the resulting participation rate dynamics as the number of players increases. The interaction of a large number of players leads to complicated participation rate patterns and our aim is to check whether the average participation rate along these patterns is consistent with the symmetric Nash equilibrium and the existing experimental and computational evidence. We also investigate how the aggregation of agents' interaction is affected by the presence of agents choosing rules that try to exploit the linear autocorrelation structure of the past participation rates. Agents who try to exploit past inefficiencies of the aggregate to improve their performances, act as some kind of arbitrageurs and we investigate how their behavior affects the properties of the system.

1.2.3 Informational differences in an asset market

Asset markets, involving an extremely large number of investors of different characteristics, are a plausible context for modeling the interaction of heterogeneous boundedly rational agents. The failure of the traditional representative rational agent framework to replicate properties of asset returns, such as excess volatility, clustered volatility, correlations of returns, persistent deviations from fundamental values (see e.g. Shiller, 1989 and Fama and French, 1988a, 1988b), explains why most of the research in the area of bounded rationality and heterogeneity has been pursued in the context of financial markets as surveyed in Hommes (2006), LeBaron (2006) and Kirman (2006).

In Chapter 4 we study a market for a financial asset populated by boundedly rational agents and we concentrate on the role of informational differences. The starting point is an asset pricing model in which agents can choose among two different degrees of information on fundamentals. At the same time agents are also learning the growth rate of the dividend generating process. An inherent feature of our model is that it contains two important benchmarks as special cases. When both informational differences and learning are both discarded, our results coincide with those of the classical Gordon model (see.g. Gordon, 1962). When only informational differences are discarded our model coincides with the one of Barsky and De Long (1993).

After developing and analyzing the full model, we investigate the extent to which our model is able to explain empirical properties of asset prices. In particular we aim at offering theoretical support to the empirical evidence that the log price of a financial asset is the sum of a persistent component and a nonlinear temporary component, which switches between two different regimes. The empirical evidence for this so-called nonlinear mean reversion is documented e.g. by Gallagher and Taylor (2001) and Manzan (2003).

Chapter 4 is also closely related to the work on informational efficiency by Grossman and Stiglitz (1980). They investigate whether the price is informationally efficient in a repeated market for a single period living asset, in which agents can decide between two different degrees of information about the value of the asset return at the end of the period. They assume that both informed and uninformed agents are rational. In a financial market where agents face informational differences, the use of rational expectations poses puzzling consequences. In fact, when the information costs are positive, if agents had rational expectations the price would fully reveal the available information about future dividends and nobody would pay for information anymore. This implies that the fraction of informed agents would go to zero. However, in the limit the price would not contain information about the dividend anymore, so that it would pay to buy information again. The absence of a rational expectation equilibrium has been referred to as the Grossman-Stiglitz paradox. In a framework with rational agents one needs two sources of noise to solve the paradox. One source is the presence of noise traders who provide liquidity to the market, the other is a noisy dividend signal for the informed agents. In this case, the model of Grossman and Stiglitz leads to a *static equilibrium degree of disequilibrium*, where agents' fractions and price distribution are constant over time and a function of the exogenous noise parameters.

In Chapter 4 we analyze the case where each agent can decide whether or not to be informed about next period's dividend, but we relax the assumption of rationality. We also endogenize the dynamics of the fraction of agents choosing to buy costly information or to extract information about future dividends from the price. We investigate whether the interaction of boundedly rational agents can offer a different solution to the Grossman-Stiglitz paradox. We argue that the interaction of boundedly rational agents, triggered by informational differences, can act as a source of *endogenous noise* to the price dynamics and no other source of noise needs to be added to the system to obtain a well-defined price and well-defined fractions. Our dynamic approach aims at offering a *dynamic equilibrium degree of disequilibrium*, in contrast with the static case of Grossman and Stiglitz. Our dynamic case is strictly connected to the endogenous noise created by agents' switching between being informed and free riding on public information.

1.2.4 Computational tools for nonlinear dynamics

Whereas in economic models with rational agents there is an emphasis on equilibria, in models with bounded rationality and heterogeneity there is an emphasis on dynamics. Since most of these dynamical systems are nonlinear, the theory of nonlinear dynamical systems is an important tool of analysis. For this reason nonlinear dynamics has become a widely used instrument in recent years. Day (1994), Gandolfo (1997), and Medio and Lines (2001) are, among others, introductory textbooks with a particular emphasis on nonlinear economic dynamics. Mathematical textbooks are e.g. Guckenheimer and Holmes (1991) and Wiggins (1990). For the convenience of the reader, we here include a brief discussion of some tools from nonlinear dynamics which will be used in our study.

In this thesis, we model agents' interaction and decision making taking place at discrete times separated by a conventional time unit, called one period, and we focus on discrete time dynamical systems. Typically, one of our model equations is the expectational feedback map related to the equilibrium pricing condition, and the others are the updating rules for the fractions of agents using different decision strategies. Whereas the equilibrium pricing equation is linear in the fractions of agents, the fact that these fractions are endogenously determined yields strong nonlinearities.

Occasionally, we investigate the effect of random exogenous shocks upon the dynamics. In this case, we will refer to the original, noise-free, system as the deterministic skeleton. Whereas the impact of noise in linear systems is well understood, results concerning the effect of noise on the local and global dynamics of nonlinear systems is limited, and therefore we have to rely on simulations.

Once a system of difference equations is derived from agents' decision making and from fractions' evolution, we typically proceed as follows. First, we search for the steady state(s) of the system. The steady state(s) of our systems typically corresponds to rational behavior. After a steady state is detected, we use local stability analysis to specify for which parameters values, the interaction and adaptation of boundedly rational agents converges to it. If a steady state is unstable, we continue with the analysis of the global dynamics and investigate the occurrence of periodic or complicated chaotic fluctuations. Whereas the dynamics in linear systems can only converge to a steady state or diverge to infinity (except hairline cases), the dynamics of nonlinear systems is richer. In our examples, the dynamics can be characterized either by convergence to a stable cycle or by irregular fluctuations and is always bounded. In particular the development of chaos theory has pointed out that deterministic dynamical systems can generate erratic time series with ongoing fluctuations whose patterns resemble those of random time series. We will encounter many of these time series in the following chapters.

In this thesis, when possible, the global dynamics is characterized analytically. However, often this is difficult or impossible, and we have to use computational tools. A useful numerical tool for detecting changes in the long run dynamics as one parameter of the model changes, is the bifurcation diagram. In a bifurcation diagram a parameter is varied and, for a grid of parameter values, the system of equations is simulated and the resulting long run behavior plotted. In such diagrams one can see that for some parameter values the state variable, say the price, converges to an equilibrium value, whereas for other parameter values the state variable oscillates along a two cycle, or follows a more complicated path. An example of a bifurcation diagram is given in the left panel of Figure 1.1, which is taken from the analysis of participation games in Chapter 3. The horizontal axes indicates the parameter N, which gives the number of agents playing the game. The vertical axes represents the participation rate, i.e. the fraction of the entire population that decides to go to the bar. The bifurcation diagrams allows to compare the dynamics of the system for different values of the parameter N. One can notice that when N is small, e.g. $N = 50$, the long run behavior of the system convergences to a participation rate of 0.5. As the number of agents N increases, the long run behavior

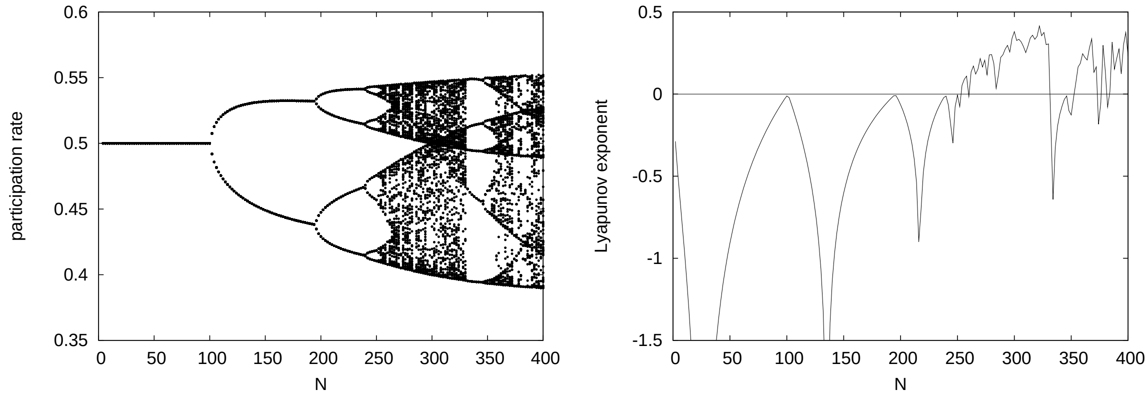

Figure 1.1: *Simulation of the dynamical systems analyzed in Chapter* 3. *Left panel: the bifurcation diagram with respect to the number of participants N gives the long run behavior of the participation rate as N changes. Right panel: Lyapunov exponents for different values of N. For every value of N,* 100 *iterations are used after a transient period of* 100.

of the interaction is more complicated. For example, when $N = 150$, the participation rate is oscillating between two different values. As N increases further other bifurcations occur and the participation rates follows cycles of period four, eight ... For future reference, this particular bifurcation structure with period of the cycle doubling step by step is called period-doubling bifurcation route to chaos. The bifurcations continue until we reach a point, $N \approx 250$, where many different values between 0.4 and 0.55 are possible.

A useful tool for analyzing whether the black region of the bifurcation diagram corresponds to a cycle of very large period or to more complex chaotic behavior is the largest Lyapunov exponent plot. Lyapunov exponents are used to characterize sensitive dependence on initial conditions, that is, whether a small change of the initial condition can lead to a large change of the realized state variable. When the system has a positive Lyapunov exponent there is sensitive dependence on initial conditions and the dynamics is chaotic. The right panel of Figure 1.1 gives an example of a Lyapunov exponent plot for the parameter corresponding to the bifurcation diagram. The horizontal axes indicates the parameter N. The vertical axes represents the Lyapunov exponent. When the Lyapunov exponent is negative, e.g. for $N = 50$ or $N = 150$, the system converges to a regular attractor, and, in fact, the bifurcation digram shows long run convergence to a stable cycle. When the Lyapunov exponent is positive, e.g. $N \approx 300$, the corresponding long run behavior is instead chaotic as suggested by the bifurcation diagram at the same value of N. These numerical tools will be used throughout the thesis to investigate global dynamics.

1.3 Negative expectational feedback

An interesting feature of the different economic frameworks analyzed in Chapters $2-4$ is that they all share the same type of feedback between agents' expectation of a variable, e.g. price, and its realization. In general, one can classify this expectational feedback structure as negative when positive (negative) deviations of the expectations from the expectational equilibrium price result in negative (positive) deviations of the realized price from the expectational equilibrium price. Conversely, the expectational feedback is positive when positive (negative) deviations of the expected price from the expectational feedback equilibrium result in positive (negative) deviations of the realized price from the expectational feedback equilibrium. When the expectational feedback map is differentiable, negative (positive) feedback corresponds to a negative (positive) first order derivative of the map around the expectational equilibrium. It turns out that all our examples show a *negative expectational feedback* structure.

The cobweb model of Chapter 2 is perhaps the best known example of a system with negative expectational feedback. If a producer expects a high (low) price, his optimal decision is to produce a high (low) quantity that will clear the market at a low (high) price, "opposite" to the producer's prediction. Participation games investigated in Chapter 3 also have this characteristic. Consider the *El Farol* bar problem as a concrete example. If most (few) agents believe that many agents will go to the bar, few (most) will show up, contradicting the majority belief. Finally, financial markets where agents have informational differences, as studied in Chapter 4, are also systems with negative expectational feedback. In fact when many agents believe that the information is valuable and buy it, an investor is better off not buying information as the information will be revealed by prices. On the other hand if nobody buys information, believing that it is better to extract the information from prices, an investor is better off buying information as there are so few informed agents that prices do not accurately reveal information. Our three models share the same expectational feedback structure because they all stem from essentially the same economic framework: repeated competition for a limited resource. This limited resource is the demand for crop or cattle in the cobweb model, the number of seats in the *El Farol* game, and the information about profitability of the listed firms in a financial market.

The opposite type of expectational feedback, positive feedback, is characterized by consistency of actions and beliefs. Coordination games are one example of positive feedback since when most agents think other player are going to play a certain action they play the same action thus creating consistency between beliefs and realizations. In general, probably both types of feedback play a role. For example, real financial markets have a negative feedback component due to informational differences, and a positive feedback component due to the fact that agents' demand is increasing in the expected (or anticipated) prices. Fashion cycles are also systems with mixed feedbacks, with agents copying each other at certain stages but also moving to different products at other moments.

A pair of concepts related to the expectational feedback are "strategic substitutability" and "strategic complementarity". These concepts were first developed in studies of firm interactions in Bulow, Geanakoplos, and Klemperer (1985), but later extended to the interaction of economic agents with bounded rationality (see e.g. Haltiwanger and Waldman, 1985). Strategies are substitutes if agents have an incentive to do the opposite of what most other players are doing as happens to be the case in systems with negative feedback. Strategies are complementary if agents have an incentive to imitate each other as in systems with positive feedback.

Recent experimental studies as reported by Camerer and Fehr (2006) and Heemeijer, Hommes, Sonnemans, and Tuinstra (2006) argue that convergence to a rational expectations equilibrium is more likely in economic systems with negative feedback and less likely in economic systems with positive feedback. Theoretical results of this thesis confirm, for systems with negative feedback, that overall convergence to the rational equilibrium is on average correct. Nevertheless, we also show that boundedly rational agents' interaction and adaptation trigger ongoing fluctuations around such an equilibrium. This is consistent with other experiments of systems with negative feedbacks as reported in e.g. Hommes, Sonnemans, Tuinstra, and Van de Velden (2007) for a cobweb setting, in e.g. Sundali, Rapoport, and Seale (1995) for market entry games and in e.g. Selten, Chmura, Pitz, Kube, and Schreckenberg (2006) for route choice games. This is also consistent with excess volatility in financial markets summarized in Shiller (1989). In general, these endogenous fluctuations can be characterized as irregular cycles along which rules perform better than others in different periods of time, but no rule is "dominating" the scene for every period. In this respect our results also support the observation that within systems with negative feedback incentives work in the direction of heterogeneity, that is, agents are better off if they do not imitate each other. In fact, such incentives explain why the persistence of heterogeneity, and consequently of endogenous fluctuations, is a robust characteristic of our models.

Bibliography

Alchian, A. A. (1950). Uncertainty, evolution and economic theory. *Journal of Political Economy* **58**, 211–221.

Anderson, S., de Palma, A. and Thisse, J. (1993). *Discrete choice theory of product differentiation*. MIT Press, Cambridge, MA.

Arthur, W. B. (1994). Inductive reasoning and bounded rationality. *American Economic Review* **84**, 406–411.

Barsky, R. B. and De Long, J. B. (1993). Why does the stock market fluctuate? *Quarterly Journal of Economics* **58**, 291–311.

Binmore, K. and Samuelson, L. (1997). Muddling through: Noisy equilibrium selection. *Journal of Economic Theory* **74**, 235–265.

Brock, W. A., Dindo, P. and Hommes, C. H. (2006). Adaptive rational equilibrium with forward looking agents. *International Journal of Economic Theory*, fothcoming.

Brock, W. A. and Hommes, C. H. (1997). Rational routes to randomness. *Econometrica* **65**, 1059–1095.

Bullard, J. B. (2006). The learnability criterion and monetary policy. *Federal Reserve Bank of St. Luis Review* **88**, 203–217.

Bulow, J. I., Geanakoplos, J. D. and Klemperer, P. D. (1985). Multimarket oligopoly: Strategic substitutes and complements. *Journal of Political Economy* **93**, 488–511.

Camerer, C. F. and Fehr, E. (2006). When does 'Economic Man' dominate social behavior. *Science* **311**, 47–52.

Chow, G. C. (1989). Rational versus adaptive expectations in present value models. *Review of Economics and Statistics* **71**, 376–384.

Conlisk, J. (1980). Costly optimizers versus cheap imitators. *Journal of Economic Behavior and Organization* **1**, 275–293.

Conlisk, J. (1996). Why bounded rationality? *Journal of Economic Literature* **34**.

Day, R. H. (1994). *An introduction to dynamical systems and market mechanism. Complex Economic Dynamics, Vol I.* MIT Press, Cambridge, MA.

Diks, C. and Dindo, P. (2006). Informational differences and learning in an asset market with boundedly rational agents. CeNDEF Working paper 06-11, University of Amsterdam.

Dindo, P. and Tuinstra, J. (2006). A behavioral model for participation games with negative feedback. CeNDEF Working paper 06-10, University of Amsterdam.

Droste, E., Hommes, C. and Tuinstra, J. (2002). Endogenous fluctuations under evolutionary pressure in Cournot competition. *Games and Economic Behavior* **40**, 232–269.

Evans, G. W. and Honkapohja, S. (2001). *Learning and Expectations in Macroeconomics.* Princeton University Press, Princeton.

Ezekiel, M. (1938). The cobweb theorem. *Quarterly Journal Of Economics* **52**, 255–280.

Fama, E. F. and French, K. R. (1988a). Dividend yields and expected stock returns. *Journal of Financial Economics* **22**, 3–25.

Fama, E. F. and French, K. R. (1988b). Permanent and temporary componenents of stock prices. *Journal of Political Economy* **96**, 246–273.

Frankel, J. A. and Froot, K. A. (1987). Using survey data to test standard propositions regarding exchange rate expectations. *American Economic Review* **77**, 133–153.

Friedman, M. (1953). *Essays in Positive Economics.* University of Chicago Press, Chicago.

Friedman, M. and Savage, L. (1948). The utility analysis of choices involving risk. *Journal of Political Economy* **56**, 279–304.

Fudenberg, D. and Levine, D. (1998). *The theory of learning in games.* MIT Press, Cambridge, MA.

Gallagher, L. A. and Taylor, M. P. (2001). Risky arbitrage, limits of arbitrage, and nonlinear adjustment in the dividend-price ratio. *Economic Inquiry* **39**, 524–536.

Gandolfo, G. (1997). *Economic Dynamics.* Springer-Verlag, Berlin, 3rd ed.

Gordon, M. J. (1962). *The Investment, Financing, and Valuation of the Corporation.* Homewood, Ill.: R.D. Irwin.

Grandmont, J. M. (1985). On endogenous competitive business cycles. *Econometrica* **53**, 995–1046.

Grandmont, J. M. (1998). Expectation formation and stability in large socio-economic systems. *Econometrica* **66**, 741–781.

Grossman, S. J. and Stiglitz, J. E. (1980). On the impossibility of informationally efficient markets. *American Economic Review* **70**, 393–408.

Guckenheimer, J. and Holmes, P. (1991). *Nonlinear oscillations, dynamical systems, and bifurcations of vector fileds*. Springer-Verlag, New York.

Haltiwanger, J. and Waldman, M. (1985). Rational expectations and the limits of rationalty: An analysis of heterogeneity. *American Economic Review* **75**, 326–340.

Heemeijer, P., Hommes, C., Sonnemans, J. and Tuinstra, J. (2006). Price stability and volatility in markets positive and negative expectations feedback: An experimental investigation. CeNDEF Working paper 06-05, University of Amsterdam.

Hommes, C. (2004). Economic dynamics. In *Encyclopedia of Nonlinear Science*. Routeledge.

Hommes, C. (2006). Heterogeneous agents models in economics and finance. In *Handbook of Computational Economics*, L. Tesfatsion and K. L. Judd, eds., vol. 2, chap. 23. Elsevier, North Holland, pp. 1109–1186.

Hommes, C., Sonnemans, J., Tuinstra, J. and Van de Velden, H. (2007). Learning in cobweb experiments. *Macroeconomic Dynamics*, forthcoming.

Kirman, A. P. (1992). Whom or what does the representative individual represent? *Journal of Economic Perspectives* **6**, 117–136.

Kirman, A. P. (2006). Heterogeneity in economics. *Journal of Economic Interaction and Coordination* **1**, 89–117.

LeBaron, B. (2006). Agent based model in finance. In *Handbook of Computational Economics*, L. Tesfatsion and K. L. Judd, eds., vol. 2, chap. 24. Elsevier, North Holland, pp. 1187–1234.

Manski, C. and McFadden, D. (1981). *Structural Analysis of Discrete Data with Econometric Applications*. MIT Press, Cambridge, MA.

Manzan, S. (2003). Nonlinear mean reversion in stock prices. CeNDEF Working paper 03-02, University of Amsterdam.

Medio, A. and Lines, M. (2001). *Non-linear Dynamics. A primer.* Cambridge University Press, Cambridge.

Muth, J. F. (1961). Rational expectations and the theory of price movements. *Econometrica* **29**, 315–335.

Ochs, J. (1990). The coordination problem in decentralized markets: An experiment. *Quarterly Journal of Economics* **105**, 545–559.

Rubinstein, A. (1998). *Modeling Bounded Rationality.* MIT Press, Cambridge, MA.

Sargent, T. (1993). *Bounded Rationality in Macroeconomics.* Clarendon Press, Oxford.

Selten, R., Chmura, T., Pitz, T., Kube, S. and Schreckenberg, M. (2006). Commuters route choice behavior. *Games and Economic Behavior*, forthcoming.

Sethi, R. and Franke, R. (1995). Behavioural heterogeneity under evolutionary pressure: Macroeconomic implications of costly optimization. *The Economic Journal* **105**, 583–600.

Shiller, R. (1989). *Market Volatility.* MIT Press, Cambridge, MA.

Simon, H. (1957). *Models of man.* John Wiley & Sons, New York.

Sundali, J., Rapoport, A. and Seale, D. (1995). Coordination in market entry games with symmetric players. *Organizational Behaviour and Human Decision Processes* **64**, 203–218.

Taylor, P. and Jonker, L. (1978). Evolutionary stable strategies and game dynamics. *Mathematical Biosciences* **40**, 145–156.

Tversky, A. and Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science* **185**, 1124–1131.

Weibull, J. (1995). *Evolutionary Game Theory.* MIT Press, Cambridge, MA.

Weibull, J. W. (1994). The 'as if' approach to game theory: Three positive results and four obstacles. *European Economic Review* **38**, 868–881.

Wiggins, S. (1990). *Introduction to applied non-linear dynamical systems and chaos.* Springer-Verlag, New York.