Optimization and decision problems

Ramsey growth model, lakes and oil fields - 31 March 2011

CompTools - Paolo Pellizari

- As you know, optimization is about the best possible outcome. You can think that it's about taking a decision, selecting the proper values of some decision variables.
- Hence, there must be a clearly stated and definied objective function.
- Think to decision as actions: they produce two effects.
  (1) (Immediate) payoff
  (2) Change of the (future) state of the world, i.e., a new situation to be faced.

  Hence, think that a decision problem is about taking the best action a among the choices in a set A of possible actions. At the same time, there is a state of the world s in some set S of possible state of the world that is affected by your decision.
- In a decisione problem, you have to take one action a_0 in state s_0; this brings you to a new state s_1 in which you do a_1, going to s_2 where you do a_3 and so on. In a diagram

  $$\text{(Begin at } s_0\text{)}\ a_0 \hookrightarrow s_1 \hookrightarrow a_1 \hookrightarrow s_2 \hookrightarrow a_3 \hookrightarrow \dots$$

  Above, profits are collected at every arrow and the objective is to maximize the "sum" of them all.
- (Example 1: unprotected sex). Example 2: Ramsey growth model.

Action	Consumption C_t
State	Capital K_t
Evolution	$K_{t+1} = K_t + Y_t - C_t$
Objective	Intertemporal utility maximization

- Lake example: you rent a lake for T years, initially $F_0 = 10$ is the quantity of fish.

  The action is the extracted amount Q, the state is the residual quantity F in the lake, evolving as

  $$F_{t+1} = (1 + r)F_t - Q_t,$$

  that reads as "the quantity inside the lake will be the previous quantity times a growth factor less the amount fished".

  The objective is the maximization of the sum of the (discounted) profits "price × quantity" at time $t = 0, \dots, T - 1$. We assume, for simplicity, a unit price and a unit discount factor β so that only quantity matters.

  We can impose several constraints, like the requirement that the terminal quantity is equali to the initial one, $F_T = F_0$, or limitations of on the fished amounts, $Q_t \le 5$.

  Now have a look at the spreadsheet at `http://virgo.unive.it/paolop/comptools.html`
- Oil field: you rent an oil field for T years, initially with Oil_0 tons of oil.

You must decide how much to extract, Q_t, for all times. This is (your) action. The state is the residual amount of oil, that evolves as $Oil_{t+1} = Oil_t - Q_t$, which reads as "in $t+1$ the oil left is what I had before less what I have extracted". This is the law of evolution of the state, that depends on what you do.

On a given year you earn $pQ_t - \frac{Q_t^2}{Oil_{t-1}}$ and you maximize the sum of all earnings (possibly discounted).

Remember to include the constraints that every E_t is non-negative and $E_t \leq Oil_{t-1}$, as you cannot extract more than what was left the previous year.

Now have a look a the spreadsheet at `http://virgo.unive.it/paolop/comptools.html`