

USER-GUIDE:

CONSUMER OPTIMAL CHOICE PROBLEM SOLUTION

Our user-guide is structured in three parts: the first part shows a graphic solution to the problem, the second part provides for a solution through a system of linear equations, the third part shows the solution obtained by standard microeconomics computations in order to show clearly every single step and the last part lists the most common mistakes made in solving our problem.

The problem is 5.11(a) p.196 B&B:

Find the optimal consumption basket given the following utility function $u(x,y) = x^2y$ constrained by income $I = 240$, price of x : $P_x = 8$ and price of y : $P_y = 2$. $MU_x = 2xy$, $MU_y = x^2$.

We can find the two marginal utilities also with R by using the following commands:

```
> z <- expression(x**2*y)
> MUx <- D(z, "x")
> MUy <- D(z, "y")
> MUx
2 * x * y
> MUy
x^2
```

According to the consumer theory the budget line can be defined as

bl: $(P_x)x + (P_y)y = I$

1. GRAPHIC SOLUTION

The optimal consumption basket is the point in which the budget line is tangent to utility curve, in order to maximize the level of utility.

First we define the utility curve, which is a function of two variables, in R:

```
u <- function(x,y) (x**2)*y
```

in order to graph a function of two variables we have to define the values of x and y , which are in this case sequences of values as follows:

```
x <- seq(0, 30, len=30)
y <- seq(0, 30, len=30)
```

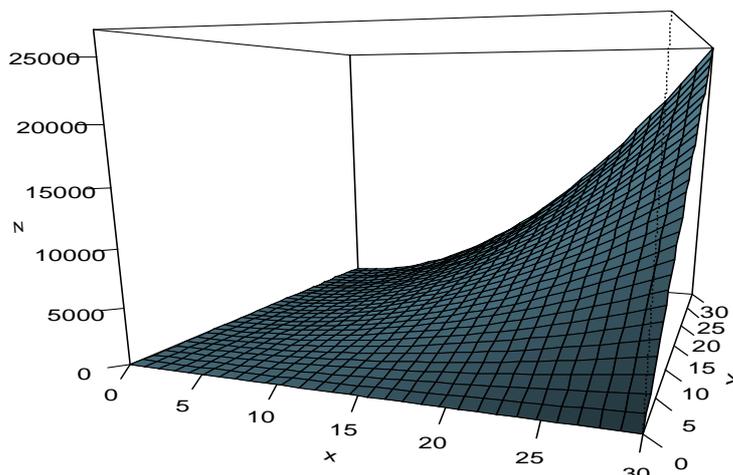
we define the third dimension z which is the result of the computation of all the possible combinations of x and y :

```
z <- outer(x,y,u)
```

to show this curve in 3D graph is necessary to use the following R-function:

```
persp(x,y,z,col="skyblue",shade=T,theta=20,phi=10,ticktype="detailed")
```

This way we can observe the behavior of the utility function which depends on the two variables x and y . On the z axis is shown the level of utility given by the combinations of x_s and y_s .



In order to see the tangency between the utility function and the budget line we have to analyze a two dimensional graph, this implies that we have multiple indifference curves whose level of utility increases as they shift away from the origin. To represent the single indifference curves we use R-function “*contour*”.

After changing the length of the sequences of x and y because of precision, we also add a grid and the axes in order to have a more precise graph.

```
x <- seq(0,30,len=200)
y <- seq(0,130,len=200)
z <- outer(x,y,u)
contour(x,y,z,col="purple",xlab="X",ylab="Y")
grid(col="grey")
text(8,120,col="purple","u(x,y)")
abline(h=0,col="grey")
abline(v=0,col="grey")
```

Then we define the budget line as a function of one variable x and graph it using R-function “*curve*” or R-function “*abline*”.

```
bl <- function(x) (240-8*x)/2
curve(bl, add=T, lwd=2, lty=2, col="red")
text(3,90,col="red","budget line")
```

or

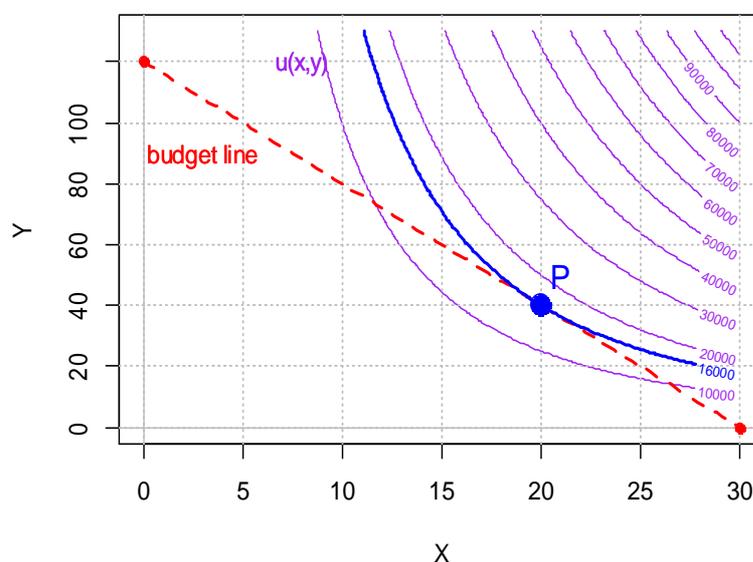
```
abline(120,-4,lwd=2,lty=2,col=2)
text(3,90,col="red","budget line")
```

From the calculations done in the second part we can calculate which is the level curve tangent to the budget line: $x^2y=16000$ and we add it on the graph.

```
contour(x,y,z,add=T,level=16000,lwd=2,col="blue")
```

It’s easy to notice that the tangency point is **P (20,40)** and we can add it to the graph. We also add to the graph the points coming from the intersection between the budget line and the axis. For all of them we use:

```
points(20,40,pch=19,cex=2,col="blue")
text(21,50,col="blue","P",cex=1.3)
points(0,120,col=2,pch=19)
points(30,0,col=2,pch=19).
```



2. SOLUTION OF THE LINEAR SYSTEM OF EQUATIONS

In order to find the optimal consumption basket we need to find the intersection between the budget line and the utility curve with maximum utility level within the budget line constraint. Since the optimal point is the tangency point between the two curves, in P the slope of the budget line $\frac{p_x}{p_y}$ is equal to the slope of the utility curve $\frac{MU_x}{MU_y}$ and we calculate this equality in a system with the budget line. Solving a system with these two conditions therefore gives us the optimal point.

A linear system in R is solved using a matrix “ $Ax=b$ ” as follows:

```
a <- matrix(c(-4, 2, 8, 2), 2 2, byrow=T)
b <- c(0, 240)
solve(a,b)
[1] 20 40
```

From the previous calculations we notice that the optimal point P is (20,40).

3. SECOND METHOD

According to the microeconomics a consumer optimization problem is solved respecting 2 conditions:

Tangency condition $\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$ subject to the budget line.

We calculated the tangency condition and we found that $y=2x$ and $x=1/2y$.

First we substitute y in the budget line equation

```
bl <- function(x,y=2*x) 8*x+2*y-240
```

And then we find the root of the equation using the R function “uniroot” in order to get the value of x in the optimal point

```
uniroot(bl,c(1,50))$root
$root
[1] 20
```

Since R by default calculates x we have to invert the label of x and y in order to get the value of y as follows and then we calculate the root of the equation

```
bl2 <- function(x,y=1/2*x) 2*x+8*y-240
uniroot(bl2,c(1,50))$root
$root
[1] 40
```

4. COMMON ERRORS

The most common mistakes made while graphing a function are: to forget to define the variable “z” using the R function “outer” and to forget “add=T” which allows to see different functions in the same graph. While solving the system instead sometimes people forget to order clearly the values plugged in the matrix and forget to write “byrow=T” since R by default reads matrices by columns.

NOTES

theta = perspective angle w.r.t. the vertical axes
shade = color shade
phi = inclination angle w.r.t. the horizontal axes
ticktype = to label ticks on the axes
cex = character expansion

lwd= line width
lty= line type
\$= extraction of one element
pch= point character
xlab,ylab= axes label